The Khazzoom-Brookes Postulate and Neoclassical Growth*

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In a disturbing assault on intuition and conventional wisdom, Khazzoom and Brookes have asserted that energy efficiency improvements might increase, rather than decrease energy consumption. If true, policies aimed at encouraging conservation could worsen rather than ameliorate global warming and would accelerate the need for offshore drilling rather than provide a substitute for it. More generally, this result would pit conservation against environmental goals, in direct contradiction to many countries' energy plans (which see conservation as an environmental solution).

Yet neoclassical growth theory confirms this possibility given certain fairly reasonable conditions—conditions that recent work by Hogan and Jorgenson indicates may hold in the U.S. economy. By no means proving the postulate, this analysis appears to make it much more difficult to dismiss.

In fact, the effect can be more dramatic than even Khazzoom and Brookes may appreciate. Energy efficiency gains can increase energy use even more directly by increasing the economic growth rate, not only by decreasing the effective cost of energy. Efficiency gains for other factors (capital and labor) can also increase energy use.

INTRODUCTION

Common sense says that energy efficiency gains will reduce energy demand below where it otherwise would be. So evident is this that most

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countries' energy policies—not to mention oil industry forecasts and many academic writings—take it as a cornerstone fact. In the way of glaring examples, the United States' National Energy Plan is not alone.

Some time ago, however, Daniel Khazzoom (1980, 1987, 1989) advanced a proposition that greater energy efficiency could lead to increased, not decreased energy demand. Amory Lovins (1988), acknowledged champion of "soft technologies," took vociferous issue with his conclusion, dismissing any non-trivial effect as fallacious. More recently, Brookes (1990) has once again raised the contention in response to widespread discussion of the greenhouse effect, claiming that the energy efficiency solution is fundamentally flawed because "reductions in energy intensity of output that are not damaging to the economy are associated with increases, not decreases, in energy demand."[1] Grubb (1990) takes issue with Brookes, in something of a throwback to the Khazzoom-Lovins debate.

Khazzoom based his contention on price elasticity arguments, focusing mainly on the household electric appliances sector. (Critics of Khazzoom sometimes seem to miss the point that energy efficiency gains in any one sector have energy consumption ramifications extending economy wide; and economy-wide impacts can get strange, as we shall see.) Brookes takes a somewhat more macroeconomic view than Khazzoom, advancing a well-articulated qualitative thesis. At the root of both arguments is the notion that energy efficiency gains look to the user a lot like price reductions, spurring increased demand either directly through price elasticity effects or indirectly through released purchasing power redirected to energy-using goods and services.

This paper takes the macroeconomic approach to the extreme and asks what neoclassical growth theory[2] might have to say about the issue. Neoclassical growth theory allows examination of factor use, factor efficiency, and factor growth dynamics at a very aggregate level, across all sectors, and over time horizons of several decades. The results, which were a surprise to me, seem to provide considerable support to the Khazzoom-Brookes postulate and possibly even extend its reach.

RE-CASTING THE "AEEI" CONTROVERSY

The Khazzoom-Brookes postulate has a direct bearing on the controversy triggered by Manne and Richels' (1990) article on the economic

1. Both Khazzoom and Brookes credit the 19th century economist Jevons with the original observation that efficiency gains could increase consumption (see Jevons, 1865).

2. Of course, the standard model of neoclassical growth is due to Solow (see, for example, Solow (1956, 1988)).

costs of CO_2 emission limits. In that article, Manne and Richels showed that the value they choose for their "autonomous (non-price-induced) energy efficiency index (AEEI)" has a dramatic impact on the estimated societal economic cost of reducing CO_2 emissions over the next century—the lower its value, the higher the cost. Many authors disputed the quantitative value chosen for the AEEI index, but none questioned that a positive AEEI would reduce energy use relative to a zero value. Moreover, all assumed, with Manne and Richels, that different values of the AEEI would have no effect on the economic growth rate.

In contrast, the Khazzoom-Brookes postulate would say that a positive AEEI would increase energy use relative to a zero value. Furthermore, the results shown below indicate that a positive AEEI may increase the economic growth rate, further increasing energy use. If true, these results would turn the AEEI controversy upside down—higher values would be associated with higher, not lower, CO_2 emissions (and therefore with higher costs of reducing CO_2 emissions).

Some argue that this stark divergence in results is symptomatic of a deeper problem. They say that the practice of specifying simulation parameters and holding them constant, when in fact their magnitude varies with the outcome, is unfortunately widespread and has been the source of many confusing results about society's ability to deal with global warming.[3]

NEOCLASSICAL ENERGY GROWTH—WITHOUT EFFICIENCY GAINS

Both the Khazzoom-Brookes postulate and the AEEI controversy take as given fixed real energy price. Of course there is no real dispute that real energy price increases result in energy efficiency gains. Rather, the AEEI controversy concerns itself with "autonomous" (i.e., non-price-induced) energy efficiency gains and the Khazzoom-Brookes postulate concerns itself with gains that are not "damaging to the economy." Both of these mean fixed energy price, and we preserve that "given" here.

Neoclassical growth theory seems a sensible way to re-cast the AEEI question and explore the Khazzoom-Brookes postulate. With it, efficiency gains can be clearly and unambiguously defined as various forms of technical progress. Most importantly, the growth dynamic and its impact on energy growth *given* efficiency gains is made explicit.

To build the argument easily, it will prove useful to begin with a kind of "base hypothesis" that ignores efficiency gains:

3. My thanks to the referee for drawing my attention to this broader concern.

Hypothesis: With fixed real energy price and with no energy efficiency improvements, energy in the long run grows in lock step with economic growth.

This hypothesis can be motivated somewhat by a (too simple) neoclassical growth model that includes energy as a factor of production, but ignores technical progress. The neoclassical growth model is readily extendible to include energy when energy price is fixed and no technical progress is assumed,[4] as shown in the Appendix. When this is done:

a) energy (E), capital (K), labor (L), and real output (Y) all grow at the same rate.

b) by implication, the E/Y ratio stabilizes and energy intensity stays fixed.

c) the real wage rate and real returns to capital, like real energy price, become fixed.

d) real consumption per worker (growth theory's welfare measure) stays fixed.

These conclusions can be illustrated with a simple quantitative model,[5] using a production function similar to Manne and Richels' and run over the same time period (see Table 1).

Table 1. Comparison of Factor Growth Rates and Productivities Given No Technical Progress

IN THE YEAR 2100:

	Annual Growth Rate (%/year)				Annual Growth Rate (%/year)				Fraction of 1990 Value	
	$\dot{Y}$	$\dot{K}$	$\dot{L}$	$\dot{E}$	$\dot{w}$	$\dot{Pe}$	$\dot{mpk}$	$(\dot{C/L})$	E/Y	C/L
Year 2100	3.0	3.0	3.0	3.0	0.0	0.0	0.0	0.0	1.0	1.0

w = real wage rate
Pe = real price of energy
mpk = real marginal productivity of capital
C = aggregate consumption of goods and services
C/L = consumption per worker

Note: These results assume a "natural" growth rate of labor of 3%/year.

4. Solow (1974) also extended the neoclassical growth model to include exhaustible resources, though his purpose was different from ours here.

5. Available from the author.

Accordingly, if one is willing to entertain a large degree of simplification, it is possible to find a measure of support for the hypothesis.

NEOCLASSICAL ENERGY GROWTH—WITH EFFICIENCY GAINS

We are now ready to examine the Khazzoom-Brookes postulate, which we might restate as follows:

Khazzoom-Brookes Postulate: With fixed real energy price, energy efficiency gains will increase energy consumption above where it would be without these gains.

(Khazzoom's statement is somewhat more restrictive than this, including conditions on price elasticity, but more about this below.)

In growth theory terms, efficiency gains are described as technical progress trends. Technical progress can enter the production function in a number of ways. For a 3-factor production function (capital, K, labor, L, and energy, E produce real economic output, Y), technical progress can be capital-augmenting, labor-augmenting, energy-augmenting, or neutral. Mathematically, this is represented as follows:

$$Y = \tau_N F(\tau_K K, \tau_L L, \tau_E E)$$

where

$$\tau_N = e^{\lambda_N t} = \text{neutral technical progress}$$

$$\tau_K = e^{\lambda_K t} = \text{capital-augmenting technical progress}$$

$$\tau_L = e^{\lambda_L t} = \text{labor-augmenting technical progress}$$

$$\tau_E = e^{\lambda_E t} = \text{energy-augmenting technical progress}$$

Technologies that improve energy efficiency will in general be a complex combination of all these forms of technical progress. Energy-augmenting technical progress could be called "pure" energy efficiency gain.

Whereas motivation for the "base hypothesis" required no assumptions about the form of the production function, a growth theory-based response to the Khazzoom-Brookes postulate does.[6] Fortunately, the main insights we are after can be developed by considering two cases:

Case 1: Cobb-Douglas Production Function

The most commonly-used form of the production function is the Cobb-Douglas form. It can be shown that,[7] for the Cobb-Douglas case (again given fixed energy price), for *all* forms of technical progress:

a) energy (E), capital (K), and real output (Y) all grow at the same rate, which is greater than the growth rate of labor (L).

b) by implication, the E/Y ratio stabilizes and energy intensity stays fixed.

c) the real return to capital, like real energy price, becomes fixed, but the real wage rate increases (labor being the increasingly scarce resource).

d) real consumption per worker grows.

and, most significantly,

e) energy consumption grows faster and becomes greater than without technical progress, and
f) real output (Y) grows faster than without technical progress.

So efficiency gains, including energy efficiency gains, increase energy consumption, contrary to intuition (or at least my intuition) and contrary to widely held beliefs about energy conservation and policies based on them. (While it may be tempting to view increased energy use as an undesirable consequence of energy efficiency gains, it should be noted that economic welfare

6. This is demonstrated in a derivation "Neoclassical Growth with Generalized Technical Progress," available from the author.
7. These results are derived in "Neoclassical Growth with Generalized Technical Progress," available from the author.

is improved by such gains, as revealed by the growth of consumption per worker.)

These results can be illustrated using the simple quantitative model modified to incorporate the four kinds of technical progress in a Cobb-Douglas production function, with each form of technical progress proceeding at 1.2%/year (see Table 2).

Table 2. Factor Growth Rates in the Year 2100 Given Various Forms of Technical Progress—Cobb-Douglas Production

	IN THE YEAR 2100:									
	Annual Growth Rate (%/year)				Annual Growth Rate (%/year)				Fraction of No Tech. Progress Case	Fraction of No Tech. Progress Case
TECHNICAL PROGRESS	$\dot{Y}$	$\dot{K}$	$\dot{L}$	$\dot{E}$	$\dot{w}$	$\dot{P}e$	$m\dot{p}k$	$(\dot{C/L})$	E	C/L
None	3.0	3.0	3.0	3.0	0.0	0.0	0.0	0.0	1.0	1.0
Neutral	4.8	4.8	3.0	4.8	1.8	0.0	0.0	1.8	6.9	5.7
Capital-Augmenting	3.5	3.5	3.0	3.5	0.5	0.0	0.0	0.5	1.7	1.8
Labor-Augmenting	4.2	4.2	3.0	4.2	1.2	0.0	0.0	1.2	3.6	4.1
Energy-Augmenting	3.1	3.1	3.0	3.1	0.1	0.0	0.0	0.1	1.1	1.1

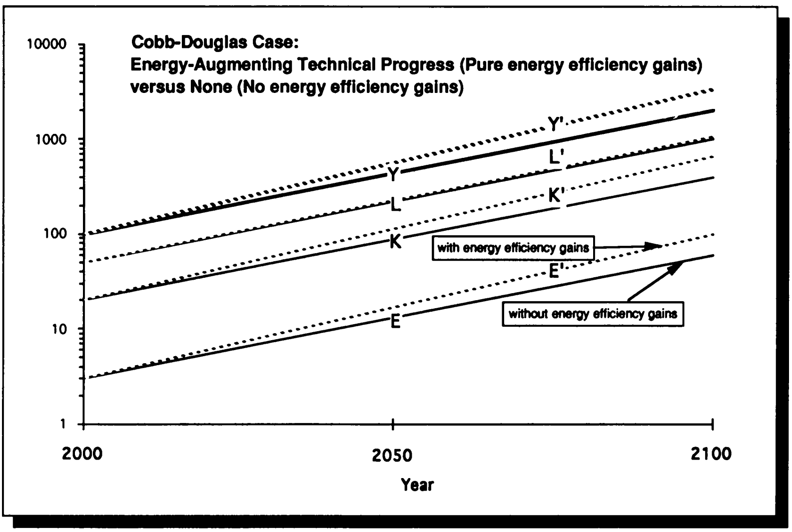
w = real wage rate
Pe = real price of energy
mpk = real marginal productivity of capital
C = aggregate consumption of goods and services
C/L = consumption per worker

Note in Table 2 how the growth rates of output, capital, and energy align. Even with technical progress, whatever its description, the base hypothesis (i.e., "with fixed real energy price. . ., energy in the long run grows in lock step with economic growth") still holds for a Cobb-Douglas production function.

Note also that all forms of technical progress increase consumption per worker (growth theory's welfare measure): returns to labor will increase since technical progress increases output by increasing the use of other factors, making labor relatively more scarce. Thus, all forms of technical progress that could be said to involve more efficient use of energy increase economic welfare.

But they also require more energy. And while it is striking that all forms of technical progress increase the absolute level of energy consumption, what is remarkable—and relevant to the Khazzoom-Brookes postulate—is that specifically, energy-augmenting technical progress ("pure" energy efficiency gain) itself increases energy use, as illustrated in Figure 1.

Figure 1. Effect of Energy Efficiency Gains on Energy Use, Output, and Other Factors (assuming Cobb-Douglas Production)



It may at first seem surprising that technical advances allowing production of output with less inputs would lead to increased use of these inputs. But consider energy, for example. Energy use grows faster for two reasons:

1. energy is effectively cheaper and thus substitutes for labor; and importantly,

2. increased economic growth due to technical progress pulls up energy use.[8]

8. I am grateful to Hill Huntington for causing me to pay more attention to this result that energy efficiency gains can increase economic growth.

In summary, therefore, if one were to accept a Cobb-Douglas production function, growth theory would then require accepting the Khazzoom-Brookes postulate.

Case 2: Nested CES Production Function

The Cobb-Douglas production function is a good starting point for generating understanding, but economists have generally found it limiting. A logical next step (and one that will generate additional insights for us) is to consider a nested CES production function, popular among energy economists.

MANNE-RICHELS NESTING

This CES (constant elasticity of substitution) function, for three factors, has a Cobb-Douglas function nested within it. The particular form favored by Manne and Richels has capital and labor combining Cobb-Douglas fashion and the two together combining CES fashion with energy[9]. For this particular nesting, designated $[(K,L),E]$, and with each form of technical progress proceeding at 1.2%/year, results are shown in Table 3.

It can be seen that capital and labor efficiency gains cause increased energy consumption (3.5%/yr and 4.2%/yr with capital-augmenting and labor-augmenting progress, respectively, compared with 3.0%/yr with no efficiency gain, or "None"), just as in the Cobb-Douglas case. Here, too, the increase comes from a combination of effects: energy substitutes for labor because energy appears cheaper and increased economic growth pulls up energy consumption. This is consistent with the Khazzoom-Brookes postulate.

The impact of neutral efficiency gains depends on the energy elasticity of substitution, σ. If σ is greater than unity, neutral efficiency gains cause increased energy consumption (5.3%/yr vs. 3.0%/yr) just like capital and labor efficiency gains cause. If σ is less than unity, neutral efficiency gains cause energy intensity to drop (energy grows at a slower rate than economic output). However, despite this more effective use of energy, the overall use of energy

9. The production function is of the form $Y = \tau_N\{a[(\tau_K K)^{\beta}(\tau_L L)^{1-\beta}]^{\rho} + b(\tau_E E)^{\rho}\}^{1/\rho}$, where neutral, capital-augmenting, labor-augmenting, and energy-augmenting technical progress factors are: $\tau_N = e^{\lambda_N t}$, and $\tau_K = e^{\lambda_K t}$, $\tau_L = e^{\lambda_L t}$, $\tau_E = e^{\lambda_E t}$, and $\rho = \dfrac{\sigma - 1}{\sigma}$, where σ is the energy elasticity of substitution.

Table 3. Factor Growth Rates in the Year 2100 given Various Forms of Technical Progress—Nested CES Production

IN THE YEAR 2100:

Annual Growth Rate (%/year)

TECHNICAL PROGRESS	$\dot{Y}$	$\dot{K}$	$\dot{L}$	$\dot{E}$		
				σ = 0.5	σ= 1.0	σ= 1.5
None	3.0	3.0	3.0	3.0	3.0	3.0
Neutral	4.7	4.7	3.0	4.1	4.7	5.3
Capital-Augmenting	3.5	3.5	3.0	3.5	3.5	3.5
Labor-Augmenting	4.2	4.2	3.0	4.2	4.2	4.2
Energy-Augmenting	3.003	3.003	3.000	2.4	3.0	3.6

σ = energy elasticity of substitution

is higher than without the efficiency gain (i.e., it grows at 4.1%/yr rather than 3.0%/yr). Again, there is consistency with the Khazzoom-Brookes postulate (irrespective of the value of σ).

However, here energy efficiency gains can lead to an apparent inconsistency with the Khazzoom-Brookes postulate. In particular, note that the effect of pure energy efficiency gains on consumption depends on the assumption used for the energy elasticity of substitution, σ. If σ is less than unity, energy consumption is reduced by the efficiency gain; if σ is greater than unity, energy consumption is increased by the efficiency gain.[10] This result is actually

10. This result is familiar to Robert Solow , who along with his reminder that factor-augmenting does not mean factor-saving, added, "it all depends on the elasticity of substitution, compared with one."

consistent with the stricter statement of the Khazzoom-Brookes postulate by Khazzoom, who added an requirement that the energy price elasticity be greater than unity. While the price elasticity and substitution elasticity are not the same thing, they are closely related, as Hogan and Manne showed.[11] A more precise statement might be that the Khazzoom-Brookes postulate holds for pure energy efficiency gains given Manne-Richels nested CES production and neoclassical growth if the energy elasticity of substitution is greater than unity. When it is less than unity, we have an exception to the broader statement of the Khazzoom-Brookes postulate.

Note that there is a key difference between this Manne-Richels CES analysis and the Cobb-Douglas analysis. There, the elasticities didn't matter, and even pure energy efficiency gains increased energy consumption by increasing economic growth. In this scheme, elasticity matters greatly to the conclusion, and energy efficiency gains do not significantly increase economic growth.

A Corollary on the Cost of Reducing CO_2 Emissions

There is an important corollary to this particular result that a high elasticity of substitution causes energy efficiency gains to boost energy consumption. On the surface, this would say that higher elasticity increases the problem of global warming. However, a higher elasticity also means that the cost of restricting energy consumption is less than it is with a lower elasticity: Hogan and Manne (1977) showed that increased energy price (as with an energy or carbon tax) has less impact on economic growth potential if elasticity is high than if it is low.

OTHER NESTING SCHEMES

But even the exception to the Khazzoom-Brookes postulate identified above depends on the particular nesting scheme used by Manne and Richels. Different nesting schemes produce contrary results. In fact, for the other two nesting schemes, that is, $[K,(L,E)]$ and $[L,(K,E)]$ vs. Manne and Richels' $[(K,L),E]$, energy efficiency gains increase energy consumption irrespective of elasticity assumptions.[12] Thus, the one exception we discovered to the Khazzoom-Brookes postulate might be relatively unique.

11. Hogan and Manne (1977) showed that the energy price elasticity equals $-\sigma/(1-s)$ for a Manne-Richels style production function, where σ is the energy elasticity of substitution, and s is the value share of energy in the economy.

12. Results available from the author.

COMBINED EFFICIENCY GAINS

To further underline this possible uniqueness, it is worthwhile reminding ourselves that this exception to the Khazzoom-Brookes postulate assumed the efficiency gain to be a pure energy efficiency gain. If more than one form of efficiency gain is at work (few technology improvements are likely to be strictly energy-specific), the situation can revert to one where the efficiency gain causes energy consumption to increase. Even for the Manne-Richels nesting scheme, a value of σ less than unity can result in increased energy consumption if any of capital, labor, or neutral efficiency gains are large enough.[13]

More General Production Functions—the Hogan-Jorgenson Results

We saw that with Cobb-Douglas production, the Khazzoom-Brookes postulate holds unambiguously, and with nested CES it holds under most conditions. How general is this result?

In an important recent analysis, Hogan and Jorgenson (1991) used a highly general (translog) production function and made empirical estimates of technology trends using a data base covering 35 sectors and spanning 20 years for the United States economy. One surprising finding is that the technical bias for energy appears to be positive. That is, with fixed energy price, Hogan and Jorgenson measure a trend of increasing value share for energy.

In our terms, this describes the expected behavior of energy-augmenting technical progress in the presence of an energy substitution elasticity greater than unity[14]. (This energy bias can be seen in Table 2, where, with fixed energy price, energy grows faster than real output for energy-augmenting technical progress and neutral technical progress. Neutral progress requires energy-specific efficiency gains. This is the only example of such a phenomenon I have found with either the Cobb-Douglas, or any CES nesting scheme.) Given our results, this suggests the presence in the U.S. economy of conditions that favor the Khazzoom-Brookes postulate—where attempts to increase the rate of energy-augmenting technical progress (i.e., to improve pure energy efficiency) will lead to increased energy consumption.

13. Further details available from the author.

14. Where with fixed energy price, we saw energy growing faster than output, while labor and capital grew at the output rate and labor and capital returns stayed fixed—a secular trend of increasing energy value share, in other words.

CONCLUSIONS

The Khazzoom-Brookes postulate says that increases in energy efficiency can lead to increased, not decreased, energy consumption. Applying neoclassical growth theory to the Khazzoom-Brookes postulate yields the following results:

1. In the absence of efficiency gains, energy use will grow in lock step with economic growth (energy intensity will stay fixed) when energy prices are fixed.

2. Energy efficiency gains can increase energy consumption by two means: by making energy appear effectively cheaper than other inputs; and by increasing economic growth, which pulls up energy use.

3. With Cobb-Douglas production, energy efficiency gains increase energy consumption in accord with the Khazzoom-Brookes postulate.

4. In fact with Cobb-Douglas production, efficiency gains for *any* factor of production increase energy consumption.

5. With the more popular nested CES production using Manne and Richels' particular nesting scheme, pure energy efficiency gains increase energy consumption if the energy elasticity of substitution is greater than unity. However, pure energy efficiency gains decrease energy consumption if the energy elasticity of substitution is less than unity. This is consistent with Khazzoom's narrower statement of the Khazzoom-Brookes postulate.

6. With nested CES production where the nesting scheme is other than Manne and Richels', energy efficiency gains increase energy consumption irrespective of elasticity conditions. This is consistent with both the broader and narrower statements of the Khazzoom-Brookes postulate.

7. Capital, labor and neutral efficiency gains increase energy consumption whether production is Cobb-Douglas or nested CES. This suggests that any technology improvement that is not strictly an energy efficiency gain may increase energy consumption.

8. Empirical results developed by Hogan and Jorgenson using a more general form of the production function suggest conditions exist that favor the Khazzoom-Brookes postulate for the U. S. economy.

These results, while by no means proving the Khazzoom-Brookes postulate, call for prudent energy analysts and policy makers to pause a long moment before dismissing it.

APPENDIX

Neoclassical Growth Model Extended to Include Energy with no Technical Progress

In this Appendix it is shown that the neoclassical growth model is readily extendible to include energy when energy price is fixed and no technical progress is assumed. The further extension to include generalized technical progress is available from the author.

Assume a constant returns to scale production function of the form[1]

$$Y = F(K, L, E),$$

where

Y = real economic output

and

K, L, and E are the factors of production:

K = Capital
L = Labor
E = Energy

1. A concern was raised by Robert Solow that with this form, nothing is said about where the "energy" comes from, especially since its production does not even absorb capital or labor. He suggested considering a two-sector model. In an analysis available from the author, it is shown that with a two-sector model of the form $Y = F[K,L,E(K,L)]$, even with technical progress terms, the conclusions in this paper about factor and output growth rates still hold. All that is required is an assumption that factors are fungible across sectors. Given this, capital and labor in the energy sector grow at the same rates they do in the economy as a whole.

Capital Growth

It is convenient to first look at this function on a "per-unit-capital" basis, rather than on the standard "per-worker" basis:

Let $\quad y = Y/K, \; l = L/K, \; e = E/K \, .$

Then $\quad Y = KF(1,L/K,E/K) = Kf(l,e) \, ,$

or $\quad y = Y/K = f(l,e) \, .$

The relative growth rate of y, designated $\dot{y}$, can be written in terms of relative growth rates of Y and K:

$$\dot{y} = \dot{Y} - \dot{K}, \quad \text{where } \dot{X} = \frac{1}{X}\frac{\partial X}{\partial t} \, . \tag{1}$$

Now presuppose the existence of a fixed growth rate, r, for output Y:

$$\dot{Y} = \mathrm{r} \, . \tag{2}$$

In standard fashion, assume net investment, I, comes via a fixed savings rate, s:

$$I = \frac{\partial K}{\partial t} = sY \, . \tag{3}$$

Rewriting (1) in terms of (2) and (3):

$$\dot{y} = r - \frac{sY}{K} = r - sy \, . \tag{4}$$

This differential equation describes a steady state only if $\dot{y} = 0$.

But y fixed means

$$\dot{y} = \dot{K} = r \, . \tag{5}$$

So for Y to grow at a fixed rate, Y and K must always grow at the same rate.[2]

Labor Growth

Now we redefine our terms to be on the standard "per-worker" basis:

Let $y = Y/L, \qquad k = K/L, \qquad e = E/L.$

Then $Y = L\,F(K/L,1,E/L) = L\,f(k,e),$

or $y = Y/L = f(k,e).$

Following the standard derivation,

$$\dot{k} = \dot{K} - \dot{L} = \frac{sY}{K} - n \ ,$$

$$\dot{k} = \frac{sy}{k} - n \ . \qquad (6)$$

For equation (6) to describe a steady state, $\dot{k} = 0$, which means

$$\dot{K} - \dot{L} = 0 \ ,$$

or

$$r = n \ . \qquad (7)$$

This means capital, labor, and output must all grow at the "natural" growth rate, n.

2. This equilibrium is stable. From equation (4), the equilibrium value of y occurring at $\dot{y} = 0$ is $y* = \frac{r}{s}$. From equation (4), if $y > y*$, $\dot{y} < 0$, and if $y < y*$, $\dot{y} > 0$.

Energy Growth

In exactly analogous fashion to labor, energy can be shown to grow at the rate n:

Let

$$y = \frac{Y}{E}, \quad k = \frac{K}{E}, \quad l = \frac{L}{E},$$

Then

$$\dot{k} = \dot{K} - \dot{E} = \frac{sY}{K} - n = \frac{sy}{k} - n . \tag{8}$$

which is in steady state only if $\dot{k} = 0$, so

$$\dot{K} - \dot{E} = 0 ,$$

or

$$\dot{E} = n . \tag{9}$$

So everything grows at the "natural rate—capital, labor, energy and output.

Equilibrium Factor Prices

Expanding Y:

$$\frac{\partial Y}{\partial t} = \frac{\partial F}{\partial K}\frac{dK}{dt} + \frac{\partial F}{\partial L}\frac{dL}{dt} + \frac{\partial F}{\partial E}\frac{dE}{dt}$$

or

$$\dot{Y} = \frac{1\partial Y}{Y\partial t} = \frac{\partial F}{\partial K}\frac{K}{Y}\dot{K} + \frac{\partial F}{\partial L}\frac{L}{Y}\dot{L} + \frac{\partial F}{\partial E}\frac{E}{Y}\dot{E} . \tag{10}$$

But, since everything grows at the "natural rate,"

$$r = \frac{\partial F}{\partial K}\frac{K}{Y}r + \frac{\partial F}{\partial L}\frac{L}{Y}r + \frac{\partial F}{\partial E}\frac{E}{Y}r . \tag{11}$$

Since all factor/output ratios stay fixed, inspection of equation (11) shows that all factor prices stay fixed over time (as do value shares).

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