A MODEL OF ASSET PRICE SPIRALS AND AGGREGATE DEMAND AMPLIFICATION
OF A "COVID-19" SHOCK

Ricardo J. Caballero
Alp Simsek

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NBER Working Paper No. 27044
April 2020
JEL No. E00,E12,E21,E22,E30,E40,G00,G01,G11

ABSTRACT

We provide a model of endogenous asset price spirals and severe aggregate demand contractions following a large supply shock. The key mechanism stems from the drop in the wealth share of the economy's risk-tolerant agents: as a recessionary supply shock hits the economy, their wealth declines and their leverage rises endogenously, causing them to offload some risky assets. When monetary policy is unconstrained, it can offset the decline in risk tolerance with an interest rate cut that boosts the market's Sharpe ratio. However, if the interest rate policy is constrained, new contractionary feedbacks arise: recessionary supply shocks not only feed into reduced risk tolerance but also into further asset price and output drops, which feed the risk-o¤ episode and trigger a downward loop. When pre-shock leverage ratios are high, multiple equilibria are possible, including one where risk-tolerant agents go bankrupt. A large-scale asset purchases (LSAPs) policy can be highly effective in this environment, as it reverses the downward asset price spiral. The Covid-19 shock and the large response by all the major central banks provide a vivid illustration of the environment we seek to capture.

Ricardo J. Caballero
Department of Economics, E52-528
MIT
50 Memorial Drive
Cambridge, MA 02142
and NBER
caball@mit.edu

Alp Simsek
Department of Economics, E52-552
MIT
50 Memorial Drive
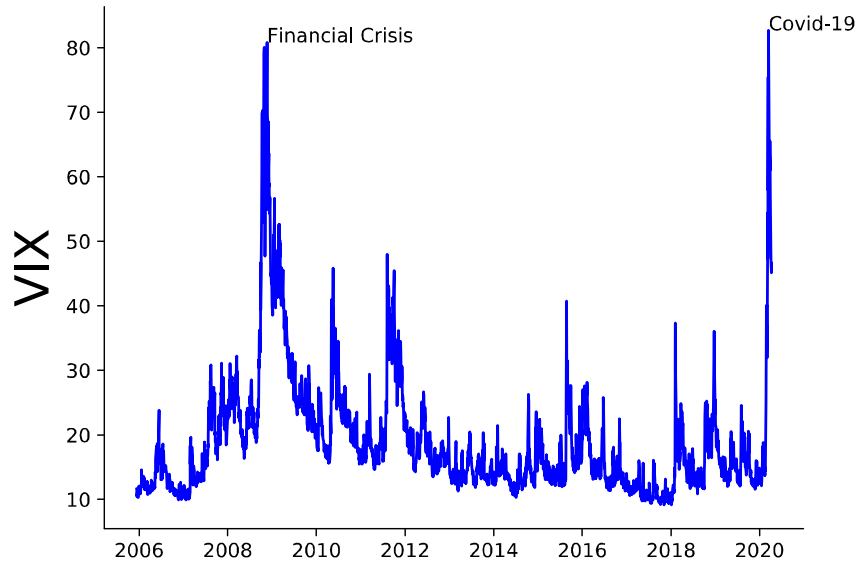Cambridge, MA 02142
and NBER
asimsek@mit.edu

Figure 1: CBOE Volatility Index (VIX)

1. Introduction

The Covid-19 shock is primarily a real (non-financial) shock that reduced aggregate supply (potential output). However, the shock also generated a large reaction in financial markets that had the potential to significantly depress aggregate demand (spending by households and firms) beyond the decline in supply. Figure 1 illustrates that the (perceived) stock market volatility spiked to levels comparable to the global financial crisis of 2008-2009. Other indicators of financial distress exhibited similar patterns—e.g., investment grade and high yield spreads tripled, and the S&P 500 dropped by 30% in a matter of weeks (a drop, per unit time, larger than the worst drop during the Great Depression). The Fed (with the backing of the Treasury) had to pledge close to 20% of US GDP in funding for a wide range of credit and market supporting facilities to stop the free fall.[1] Central banks

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[1] Here is a brief chronology of the Fed's main policy actions since early March until April 9th: On 03/03, implements a 50bps emergency rate cut; on 03/12, adds repo of up to $500b/week, purchases wider range of securities under current $60b/month program; on 03/15, cuts rates by 100bps to zero and initiates QE bond buying program of $700b, lowers swap lines with major central banks by 25bps; on 03/17, establishes a commercial paper funding facility to provide stability to short-term CP market; on 03/19, launches USD liquidity-swap lines with a broad range of countries, including major Emerging Markets; on 04/09, implements $2.3t emergency measures, among them a $500b Municipal Liquidity Facility for state and local governments, a $600b Main Street Lending program, and a Paycheck Protection Program Liquidity Facility for small businesses; expands the Primary and Secondary Market Corporate Credit Facilities and the term loan facility to buy ABS securities to $850b and includes asset purchases of HY

in the Group of Seven countries purchased $1.4 trillion of financial assets in March alone. The final story is yet to be told.

In this paper we provide a model of the amplification of large supply shocks that follows from the endogenous asset price spirals and the severe aggregate demand contractions that these spirals generate. Our model builds on a two-period version of the macroeconomic model in Caballero and Simsek (forthcoming). Briefly, that model is a variant of the New Keynesian model, but formulated in terms of a risk-centric decomposition. Specifically, there we decompose the demand block of the equilibrium into two relations: an *output-asset price relation* that captures the positive association between asset prices and aggregate demand; and a *risk balance condition* that describes asset prices given risks, risk attitudes, beliefs, and the interest rate. This decomposition facilitates the study of the *macroeconomic* impact of a variety of forces that affect risky asset prices.[2] In the current model, we extend that analysis by splitting investors into risk-tolerant and risk-intolerant agents—we dub the risk tolerant agents "banks" (interpreted broadly to include the shadow financial system and other agents able/willing to hold substantial risk) and the risk intolerant ones "households" (also interpreted broadly). The key implication of this assumption is that banks are levered in equilibrium, and therefore are highly exposed to aggregate supply shocks and the sequence of events that these shocks may trigger.[3]

To fix ideas, consider a large negative supply shock (e.g., a Covid-19 shock). This shock exerts downward pressure on risky asset prices (which include credit, equity, real estate, as well as other assets). As banks incur losses, their leverage rises. With higher leverage, banks require a higher Sharpe ratio (risk premium per unit of risk) to hold the same amount of risky assets. Risk-intolerant households also require a higher Sharpe ratio to hold the risky assets unloaded by banks wishing to reduce their leverage. Either way, the result is a rise in the market's *required* Sharpe ratio.

As a benchmark, suppose the banks' initial leverage is not too high and the supply

bonds, HY ETFs, CLOs, and CMBS securities. All other major central banks around the world have also pursued unprecedented financial markets interventions.

[2]The decomposition is suported by a growing empirical literature that shows risky asset prices can substantially affect aggregate demand. See Gilchrist and Zakrajšek (2012) on the effect of credit spreads on investment and consumption; Mian and Sufi (2014) and Chodorow-Reich et al. (2019) on the effect of house and stock prices, respectively, on consumption and (nontradable) employment; Pflueger et al. (forthcoming) on the effect of financial market risk perceptions on economic activity and interest rates.

[3]Moreover, banks' leverage and exposure can be indirect. For example, the US entered the Covid-19 shock with well capitalized (regular) banks and highly indebted corporations. However, to the extent that banks had lent to these highly levered corporations, banks themselves are highly levered with respect to large aggregate shocks. It is no accident that on 04/14/2020 JPMorgan Chase announced its highest loan-loss provision in a decade. Since then, all other major banks have made similar announcements in their earnings reports.

shock is temporary. In this case, a small decline in asset prices may be all that is needed to increase the Sharpe ratio as much as the market demands. Asset prices and aggregate demand are relatively high and the natural interest rate ("rstar") may not decline. Intuitively, supply is temporarily low but asset prices and demand *per unit of current supply* are not necessarily low, as investors expect a speedy recovery.

In contrast, we focus on scenarios in which banks' initial leverage is high (or the supply shock is sufficiently large). In this case, even a temporary supply shock greatly reduces effective risk tolerance and increases the required Sharpe ratio. This exerts substantial downward pressure on asset prices and aggregate demand, and reduces "rstar." The decline in risk tolerance overwhelms the expected recovery effect and induces a disproportionate decline in demand that exceeds the decline in supply. When the supply shock is more persistent, the downward pressure on asset prices is stronger and the (negative) gap between aggregate demand and supply becomes even greater.

The first line of defense is conventional monetary policy that cuts the interest rate (consistent with lower "rstar"). This provides the market with the greater Sharpe ratio that it requires and defuses the downward pressure on asset prices. Asset prices and aggregate demand decline in proportion to the reduction in supply but no more. However, if the interest rate is constrained, then asset prices decline beyond the reduction in supply. Lower prices provide the market with a greater Sharpe ratio but they also generate a demand recession: output falls beyond the reduction in potential output. To make matters worse, the decline in asset prices further reduces the banks' wealth share (and raises their leverage), which further increases the required Sharpe ratio and depresses asset prices, triggering a downward spiral. We show that when banks' initial leverage is sufficiently high, the feedback between asset prices and risk intolerance becomes so strong that multiple equilibria are possible. In the worst of these equilibria, banks go bankrupt.

This description of events suggests that policies where the *consolidated* government (e.g., the Fed and the Treasury in the U.S.) absorbs some of the risk that banks are struggling to hold can be highly effective. We loosely refer to these policies as large-scale asset purchases (LSAPs). We show that to the extent that the government has future *fiscal capacity*, LSAPs are powerful because they reverse the downward spirals. That is, they exhibit a high multiplier precisely when the economy is most unstable. We further show that it is optimal for the government to deploy LSAPs when the aggregate demand amplification of the supply shock is severe, even if the government is less risk tolerant than the market. In the Covid-19 episode, the spike in VIX began to reverse after the major central banks' policy actions (Figure 1 and Footnote 1), which suggests the interventions were effective in containing the initial downward spiral.

Section 2 describes the model. Section 3 shows how LSAPs operate in this environment. Section 4 concludes. Appendix A contains derivations omitted from the text.

Literature review. At a methodological level this paper adopts the risk-centric perspective in Caballero and Simsek (forthcoming, 2020). The novel ingredient is that the supply shock endogenously lowers risk tolerance. The mechanism of endogenous leverage and asset price spirals is also central in Brunnermeier and Sannikov (2014).[4] Their focus is on financial frictions on the supply side of the economy rather than on the heterogeneity of investors' portfolios and the feedback loops with aggregate demand when monetary policy is constrained. Also related is Caballero and Krishnamurthy (2009), who show how the endogenous leverage of the US economy caused by the global demand for safe assets creates instability with respect to supply shocks, but they do not discuss the role of aggregate demand and central bank policy in such a mechanism.

While they do not look at the effect of large supply shocks, the mechanism in Kekre and Lenel (2020) is close to ours. In particular, they calibrate a model in the spirit of Caballero and Simsek (forthcoming) and show the power of monetary policy in affecting the risk premium when agents have heterogeneous risk tolerance. Similarly, Caballero and Farhi (2018) show that when a large share of wealth is allocated to extremely risk-intolerant agents (Knightians) in a New Keynesian framework with a zero lower bound on interest rates, the economy may fall into a "safety trap." Like our paper, they show that asset market policies where the government absorbs part of the risk of the economy (and replace it with safe assets) can be highly effective. However, their focus is on the macroeconomic implications of a chronic scarcity of safe assets rather than on the role of endogenous risk intolerance following a large supply shock.

In terms of one of our central underlying questions, which is whether demand factors can exacerbate the direct effect of the supply shock, the closest paper to ours is Guerrieri et al. (2020). They provide a clean decomposition of the ingredients needed for an affirmative answer in a two-period, deterministic model. They conclude that, in such a model, aggregate demand cannot exacerbate the supply recession when the economy has a single sector, regardless of whether markets are complete or incomplete. In contrast, they show that in a multi-sector environment there are configurations of preference parameters where demand responds by more than supply, especially so if markets are incomplete. Our risk-based mechanism is orthogonal to theirs. In fact, our model has a single sector.

The Covid-19 shock has triggered a large response among macroeconomists. For ex-

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[4]See also Kiyotaki and Moore (1997); Shleifer and Vishny (1997); Lorenzoni (2008); Geanakoplos (2010); Adrian and Shin (2010); He and Krishnamurthy (2013); Di Tella (2017); Dávila and Korinek (2018); Cao et al. (2019).

ample, Eichenbaum et al. (2020); Faria-e Castro (2020) embed pandemic shocks and their constraints on economic activity within DSGE models and study the role of fiscal policy and different containment strategies. Baker et al. (2020) document the dramatic spike in uncertainty and study its impact in a real business cycle model. Our analysis is complementary as we emphasize the excessive aggregate demand contraction that results from supply shocks—which is exacerbated by uncertainty—and we highlight the damage caused by *the pricing of uncertainty*. Fornaro and Wolf (2020) provide a stylized New Keynesian model and capture the Covid-19 shock as a decline in (exogenous and endogenous) expected growth. Their mechanisms and policy analysis do not operate through endogenous spikes in risk intolerance and asset price spirals, which is our focus. Correia et al. (2020) use the 1918 flu pandemic to empirically analyze the economic costs of pandemics and find a role for both supply- and demand-side channels, consistent with our analysis.

There are also several papers that embed SIR type epidemiological models into macroeconomic models and study the optimal containment policy that balances health concerns and economic costs (e.g., Atkeson (2020); Alvarez et al. (2020); Eichenbaum et al. (2020); Gourinchas (2020); Baldwin (2020); Berger et al. (2020); Callum et al. (2020); Bethune and Korinek (2020)). We do not address this important trade-off and take as given the broad supply implications of the containment policies.

Finally, our policy analysis is related to a growing literature on the role of central bank asset purchases in stimulating aggregate demand when conventional monetary policy is constrained. Empirical evidence suggests these policies have a meaningful impact on asset prices (see Bernanke (2020)) but the underlying mechanisms are not fully understood. The literature emphasizes either financial frictions (e.g., Gertler and Karadi (2011); Del Negro et al. (2017)), portfolio balance effects in segmented markets (e.g., Vayanos and Vila (2009); Ray (2019)), or signaling effects (see, e.g., Bhattarai et al. (2015)). The mechanism in our paper is different and relies on the government's ability to absorb aggregate risk using its future tax capacity in a non-Ricardian model (see also Silva (2016)).[5]

2.   The Baseline Model

We present a simple two period model that illustrates how a supply shock can be amplified by an aggregate demand contraction that exceeds the supply shock itself. The mechanism operates through risk markets: The decline in asset prices due to the supply shock lowers the wealth share and increases the leverage of risk-tolerant agents. As these agents attempt

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[5]Our mechanism builds upon the extensive literature spurred by Holmström and Tirole (1998) on the taxation power of the government to expand the supply of liquidity.

to withdraw from risk, asset prices and aggregate demand drop. The demand-induced decline in asset prices further lowers the wealth share of risk-tolerant agents, and so on.

A two-period risk-centric aggregate demand model. Consider an economy with two dates, $t \in \{0, 1\}$, a single consumption good, and a single factor, capital. There is no investment or depreciation and capital is normalized to one unit. We let z_t denote the productivity of capital in period t. *Potential* output is equal to productivity, z_t, but actual output can be below this level due to a shortage of aggregate demand, $y_t \leq z_t$. We assume output is equal to its potential at the last date, $y_1 = z_1$, and focus on the endogenous determination of output at the previous date, $y_0 \leq z_0$. We assume the productivity at date 1 is uncertain and log-normally distributed,

$$\log y_1 = \log z_1 \sim N \left(\log \overline{z}_1 - \frac{\sigma^2}{2}, \sigma^2 \right). \tag{1}$$

Note that $\overline{z}_1$ captures the expected productivity, and σ captures its volatility.

There are two types of assets. There is a "market portfolio" that represents claims to all output (which accrue to production firms as earnings), and a risk-free asset in zero net supply. We denote the (ex-dividend) price of the market portfolio at date 0 with $z_0 Q_0$, so that Q_0 corresponds to the price *per unit of productivity*. We denote the log risk-free interest rate with r^f, and the log return of the market portfolio with

$$r(z_0, z_1) = \log \left(\frac{z_1}{z_0 Q_0} \right). \tag{2}$$

There are two types of agents, $i \in \{b, h\}$. Type b agents ("banks") are more risk tolerant than type h agents ("households"). Formally, agents have Epstein-Zin utility with risk aversion parameters given by $1/\tau^i$ that satisfy $\tau^b > \tau^h$. We refer to τ^i as agent i's risk tolerance. Agents also have common EIS equal to one (for simplicity), and common discount rate denoted by $e^{-\rho}$.

Agents are endowed with initial positions that satisfy:

$$\tilde{a}_0^b = \max \left(0, \overline{a}_0^b \right) \text{ and } \tilde{a}_0^h = \min \left(y_0 + z_0 Q_0, \overline{a}_0^h \right), \tag{3}$$

where

$$\overline{a}_0^b = \kappa_0 \left(y_0 + z_0 Q_0 \right) - \left(1 + e^{-\rho} \right) \kappa_0 l_0 \tag{4}$$

$$\overline{a}_0^h = \left(1 - \kappa_0 \right) \left(y_0 + z_0 Q_0 \right) + \left(1 + e^{-\rho} \right) \kappa_0 l_0,$$

for some $\kappa_0, l_0 \in (0, 1)$.

Eq. (4) describes banks' endowments and net wealth assuming they are not bankrupt. Banks initially hold a fraction of the market portfolio, κ_0, and owe $(1 + e^{-\rho}) \kappa_0 l_0$ units of safe debt. We have normalized these positions so that in *the benchmark*, defined as the case when there is no demand recession (efficient) and the supply shock is normalized to one, $z_0 = 1$, banks' leverage ratio (defined as their debt-to-asset ratio) is l_0. Households hold the mirror image positions: they hold the residual fraction of the market portfolio, $1 - \kappa_0$, as well as banks' safe debt. Eq. (3) adjusts agents' net wealth for the possibility of bankruptcy. When $\bar{a}_0^b < 0$, the value of banks' assets is less than their outstanding debt. In this case, banks are bankrupt and their actual net wealth is zero, $\tilde{a}_0^b = 0$. Households take over banks' assets. They hold all of the market portfolio so their net wealth becomes $\tilde{a}_0^h = y_0 + z_0 Q_0$.

Given the initial endowments in (3), agents choose their consumption and new asset positions, c_0^i and a_0^i, and what fraction of their assets to allocate to the market portfolio, ω_0^i, with the residual fraction invested in the risk-free asset. We formally state and solve the investors' problem in the appendix. The assumption on the EIS implies households spend a fraction of their wealth,

$$c_0^i = \frac{1}{1 + e^{-\rho}} \tilde{a}_0^i \quad \text{and} \quad a_0^i = \frac{e^{-\rho}}{1 + e^{-\rho}} \tilde{a}_0^i. \tag{5}$$

Agents' optimal weight on the market portfolio is approximately given by

$$\omega_0^i \sigma \simeq \tau^i \frac{E\left[r\left(z_0, z_1\right)\right] + \frac{\sigma^2}{2} - r^f}{\sigma}. \tag{6}$$

This is a standard mean-variance portfolio optimality condition that says the risk of agents' optimal portfolio (the left side) is proportional to the Sharpe ratio on the market portfolio (the right side). This equation holds exactly in continuous time but only approximately in discrete time. We assume the equation is exact to simplify the analysis.

Asset markets clearing condition can be written as

$$\sum_i \omega_0^i a_0^i = z_0 Q_0. \tag{7}$$

The supply side of the economy is described by New-Keynesian firms that have fixed nominal prices. These firms meet the available demand at these prices as long as prices are higher than their marginal cost. Output is determined by the aggregate demand for

goods (consumption) up to the capacity constraint,

$$y_0 = \sum_i c_0^i \le z_0. \tag{8}$$

Finally, we assume that the interest rate policy attempts to replicate the supply-determined output level, subject to a lower bound constraint, $r^f \ge 0$. Specifically, suppose the monetary policy follows a standard Taylor rule, $r^f = \max(0, \psi(y_0 - z_0))$. We focus on the limit $\psi \to \infty$, in which case this rule implies that either the interest rate is positive and output is at its potential, $r^f = r^{f*} > 0$ and $y_0 = z_0$; or the interest rate is constrained and there is a demand recession, $r^f = 0$ and $y_0 \le z_0$. Here, r^{f*} denotes the natural interest rate consistent with potential output, $y_0 = z_0$ [see Eq. (17) below].

Equilibrium characterization. We next characterize the equilibrium. Using Eq. (5), aggregate consumption is a fraction of aggregate wealth,

$$c_0 = \frac{1}{1 + e^{-\rho}}(y_0 + z_0 Q_0). \tag{9}$$

Combining this expression with Eq. (8), we obtain the following equation:

$$y_0 = e^\rho z_0 Q_0. \tag{10}$$

We refer to this equation as *the output-asset price relation.* The condition says that higher asset prices increase aggregate wealth and consumption, which leads to greater output.

Setting $y_0 = z_0$ in (10), we obtain the efficient level of asset price per productivity as

$$Q^* = e^{-\rho}. \tag{11}$$

This is the asset price per unit of productivity that ensures the economy operates at the supply determined level. If there is a supply shock that reduces z_0, asset prices should fall proportionally to $z_0 Q^*$, but no further. Any further reduction in asset prices would trigger a demand recession as illustrated by (10).

Next consider the characterization of the equilibrium asset price, $z_0 Q_0$. To facilitate this analysis, we define banks' (post-z_0-shock) wealth share as

$$\alpha_0 \equiv \frac{a_0^b}{z_0 Q_0}.$$

Households' wealth share is the residual, $1 - \alpha_0 \equiv \frac{a_0^h}{z_0 Q_0}$. Using this notation, we can write the asset market clearing condition (7) as

$$\alpha_0 \omega_0^b + (1 - \alpha_0) \omega_0^h = 1. \tag{12}$$

The equilibrium asset price is determined by this condition together with agents' wealth shares, $\alpha_0, 1 - \alpha_0$, and their optimal portfolio weights, ω_0^b, ω_0^h.

To calculate the wealth shares, we use the output-asset price relation in (10) together with agents' initial positions in (3) and their optimal saving rule in (5). For the banks' wealth share, we obtain

$$\alpha_0 = \alpha_0 (z) \equiv \max \left(0, \left(1 - \frac{l_0}{z} \right) \kappa_0 \right) \quad \text{where} \quad z = z_0 \frac{Q_0}{Q^*}. \tag{13}$$

To understand this expression, first consider the benchmark with $Q_0 = Q^*$ and the supply shock normalized to one, $z_0 = 1$. In this benchmark, $z = 1$ and banks' wealth share is given by $\alpha_0 = (1 - l_0) \kappa_0$: their initial assets net of their leverage. Now suppose asset valuations fall, $z = z_0 (Q_0/Q^*) < 1$, either because of a decline in productivity, z_0, or a decline in the asset price per productivity, Q_0. This causes banks' wealth share to fall below the benchmark (and households' wealth share increases above the benchmark). Intuitively, since banks are levered, a decline in asset valuations reduces their wealth more than it reduces households' wealth. This mechanism will play an important role for our results. If asset valuations decline beyond banks' initial leverage, $z = z_0 (Q_0/Q^*) < l_0$, banks are bankrupt and their wealth share falls to zero.

To calculate the optimal portfolio weights, we use Eq. (6) together with the expected return on the market portfolio from Eqs. $(1 - 2)$ to obtain

$$\omega_0^i \sigma = \tau^i \frac{\log \frac{\bar{z}_1}{z_0} - \log (Q_0) - r^f}{\sigma}. \tag{14}$$

Combining Eqs. $(12 - 14)$, we obtain the central equation of our analysis, *the risk balance condition*:

$$\frac{\sigma}{\tau_0 \left(z_0 \frac{Q_0}{Q^*} \right)} = \frac{\log \frac{\bar{z}_1}{z_0} - \log (Q_0) - r^f}{\sigma}. \tag{15}$$

Here, $\tau_0(\cdot)$ denotes *the effective risk tolerance* defined as

$$
\begin{aligned}
\tau_0(z) &\equiv \alpha_0(z)\,\tau^b + (1 - \alpha_0(z))\,\tau^h \\
&= \max\left(\tau^h, \tau^h + \left(1 - \frac{l_0}{z}\right)\kappa_0\left(\tau^b - \tau^h\right)\right).
\end{aligned}
\tag{16}
$$

Eq. (15) says that the risk the economy generates normalized by the effective risk tolerance (the left side) should be compensated by a sufficiently high reward for risk (the right side). Specifically, the right side is *the actual Sharpe ratio* on the market portfolio: the risk premium per unit of risk. In the rest of the paper, we refer to the expression on the left side as *the required Sharpe ratio*, and note that the equilibrium in risk markets obtains when the required and actual Sharpe ratios are the same.

Eq. (16) illustrates that the effective risk tolerance—which determines the required Sharpe ratio—depends on a wealth-weighted average of investors' risk tolerances. The second line solves for the effective risk tolerance and shows that it is increasing in $z = z_0 \frac{Q_0}{Q^*}$. In particular, a decline in asset prices—either through reduced productivity, z_0, or reduced valuation per productivity, Q_0—reduces the effective risk tolerance. Lower asset prices reduce banks' wealth share, which lowers the effective risk tolerance since $\tau^b > \tau^h$. If banks go bankrupt, the effective risk tolerance is given by the households' tolerance, $\tau_0(z) = \tau^h$.

The equilibrium is then determined by the output-asset price relation (10), the risk balance condition (15), and monetary policy.

Temporary supply shocks can reduce aggregate demand and interest rates. We next consider the comparative statics of temporary supply shocks—a reduction in z_0 keeping $\bar{z}_1$ unchanged. An example is the Covid-19 shock. In this context, we illustrate how, when banks' outstanding leverage is sufficiently high, temporary supply shocks *reduce* aggregate demand by *more* than the aggregate supply shock, and hence *reduce* interest rates.

First suppose there is no lower bound on the interest rate. In this case, monetary policy always ensures output is equal to its supply-determined level, $r^f = r^{f*}$ and $y_0 = z_0$. This requires the asset price per productivity to be at its efficient level, $Q_0 = Q^*$ [cf. (11)]. Combining this with Eq. (15), the interest rate also needs to be at a particular level,

$$
r^{f*} = \rho + \log \frac{\bar{z}_1}{z_0} - \frac{\sigma^2}{\tau_0(z_0)}.
\tag{17}
$$

10

Consider a decline in z_0 keeping $\bar{z}_1$ unchanged. Eq. (17) illustrates that this exerts two effects on the risk-free interest rate. On the one hand, a decline in z_0 increases the expected growth rate, $\frac{\bar{z}_1}{z_0}$, which increases the interest rate. Intuitively, while asset prices are currently relatively low, they are expected to recover. This raises the Sharpe ratio and induces agents to invest in the market portfolio [cf. (14)], which exerts upward pressure on the asset price per productivity. The interest rate increases to keep asset prices at the efficient level. On the other hand, a decline in z_0 also reduces banks' wealth share [cf. (13)], which decreases the interest rate. Since banks are levered, a decline in asset valuations reduces their wealth more than the households' wealth. This decreases effective risk tolerance and puts downward pressure on asset prices and the interest rate.

The second channel dominates (locally), $\frac{dr^{f*}}{dz_0} > 0$, as long as the parameters satisfy $z_0 > l_0$ (no bankruptcy) and

$$\frac{l_0}{z_0} \kappa_0 \left(\tau^b - \tau^h \right) > \left(\frac{\tau_0 \left(z_0 \right)}{\sigma} \right)^2, \tag{18}$$

$$\text{where } \tau_0 \left(z_0 \right) = \tau^h + \left(1 - \frac{l_0}{z_0} \right) \kappa_0 \left(\tau^b - \tau^h \right).$$

All else equal, temporary supply shocks are more likely to reduce aggregate demand by more than supply when agents' risk tolerance is more heterogeneous (greater $\tau^b - \tau^h$, keeping $\tau_0 \left(z_0 \right)$ constant), when banks have greater initial leverage (greater l_0), and when the shock is more severe (lower z_0). In fact, when households are very risk intolerant, $\tau^h = 0$, condition (18) is satisfied whenever the debt to productivity ratio exceeds a threshold, $\frac{l_0}{z_0} \geq \tilde{d} \in (0, 1)$.

For the rest of the paper, we isolate our leverage mechanism and simplify the analysis by assuming

$$\log \bar{z}_1 = \log z_0 + g, \tag{19}$$

where g is an exogenous growth parameter. Hence, we focus on permanent supply shocks, which simplifies the equations but it is not necessary for our results: we could have instead worked with parameters that satisfy (18). With Eq. (19), the risk-free interest rate is given by

$$r^{f*} = \rho + g - \frac{\sigma^2}{\tau_0 \left(z_0 \right)}. \tag{20}$$

Hence, in this case a decrease in z_0 always (weakly) reduces aggregate demand and the interest rate.

Supply shocks can trigger downward asset price spirals. We next consider the case where there is a lower bound on the interest rate. In this case, the supply shock *can* cause a demand recession. We assume the parameters satisfy [cf. (16)]

$$\tau_0\left(1\right) \geq \frac{\sigma^2}{\rho + g} > \tau^h. \tag{21}$$

The first inequality ensures that when supply is equal to its benchmark level, $z_0 = 1$, there is an equilibrium with an unconstrained (positive) interest rate. The second inequality ensures that, if households control all the wealth in the economy, the interest rate is constrained (zero).

Our next result characterizes the equilibrium for different levels of the productivity shock, z_0. To state the result, we define the *normalized asset price* per productivity, $\tilde{Q}_0 \equiv \frac{Q_0}{Q^*} \in [0, 1]$, which simplifies the notation. We also define two cutoffs for productivity that we denote with z^h and z^*.

The first cutoff, z^h, is the productivity level below which there is an equilibrium where banks go bankrupt and households control all wealth. To calculate this cutoff, suppose there is bankruptcy. Using the risk balance condition (15) with $\tau_0 = \tau^h$ (and $r^f = 0$), we obtain

$$\tilde{Q}^h = \frac{Q^h}{Q^*} \equiv \exp\left(g + \rho - \frac{\sigma^2}{\tau^h}\right) < 1. \tag{22}$$

Note that $\tilde{Q}^h$ is the minimum normalized asset price. Suppose the price falls to this level, $\tilde{Q}_0 = \tilde{Q}^h$. Then, Eq. (13) implies banks will indeed go bankrupt as long as productivity is sufficiently low:

$$z_0 < z^h \equiv \frac{l_0}{\tilde{Q}^h}. \tag{23}$$

When $z_0 < z^h$, there is always a bankruptcy equilibrium. Note that the cutoff z^h is increasing in l_0: bankruptcy is more likely when banks have greater initial leverage.

The second cutoff, z^*, is the productivity level above which there is a supply determined equilibrium with the efficient price $\tilde{Q}_0 = 1$. To calculate this cutoff, we use the risk balance condition (15) with $\tilde{Q}_0 = 1$ and $r^f = 0$ to obtain the value of $z^* < 1$ that solves

$$\tau_0\left(z^*\right) = \frac{\sigma^2}{\rho + g}. \tag{24}$$

When $z_0 > z^*$, there is always an equilibrium with the efficient asset price.

Proposition 1. *Consider the equilibrium with condition (21). Let z^h and z^* denote the cutoffs defined by Eqs. (23) and (24).*

(i) If $z_0 > z^h$, then the equilibrium is unique and does not feature bankruptcy. If $z_0 \in \left(z^h, z^*\right)$ (assuming the interval is nonempty), then the equilibrium features an interior asset price, $\tilde{Q}_0 \in \left(\tilde{Q}^h, 1\right)$, that solves

$$\frac{\sigma}{\tau^h + \left(1 - \frac{l_0}{z_0 \tilde{Q}_0}\right) \kappa_0 \left(\tau^b - \tau^h\right)} = \frac{g + \rho - \log\left(\tilde{Q}_0\right)}{\sigma}. \tag{25}$$

Reducing productivity reduces the equilibrium price per productivity, $\frac{d\tilde{Q}_0}{dz_0} > 0$. If $z_0 \geq z^*$ (as well as $z_0 > z^h$), the equilibrium features the efficient asset price, $\tilde{Q}_0 = 1$.

(ii) If $z_0 \leq z^h$, then there is a bankruptcy equilibrium with the low asset price, $\tilde{Q}_0 = \tilde{Q}^h < 1$. There might also be other equilibria. When $z_0 \in \left[z^*, z^h\right]$ (assuming the interval is nonempty), there is also an equilibrium with the efficient asset price, $\tilde{Q}_0 = 1$.

The first part of Proposition 1 shows that the equilibrium is unique as long as the shock is not severe enough to trigger bankruptcy ($z_0 > z^h$). In this region, when the supply shock is below a cutoff ($z_0 < z^*$), the equilibrium features a demand recession. More severe supply shocks lead to lower asset prices and more severe demand recessions. As we will see below, these supply shocks also generate downward spirals and have an amplified effect on asset prices and aggregate demand. The second part of Proposition 1 shows that, when the shock is sufficiently severe to trigger bankruptcy ($z_0 < z^h$), these amplification mechanisms can lead to multiple equilibria.

To illustrate the first part of Proposition 1, consider the case with an interior equilibrium price. Substituting $r^f = 0$ and $\tilde{Q}_0 = Q_0/Q^*$ in the risk balance condition (15), we find that the price solves Eq. (25). This equation has a natural interpretation. The right side is *the actual Sharpe ratio* (with constrained interest rate $r^f = 0$). It is decreasing in $\tilde{Q}_0$: lower asset prices increase the risk premium and the Sharpe ratio. The left side is *the required Sharpe ratio* (assuming there is no bankruptcy). It is also decreasing in $\tilde{Q}_0$: lower asset prices transfer (relative) wealth from banks to households, which reduces the effective risk tolerance and requires a greater Sharpe ratio for agents to absorb the risk. Figure 2 illustrates these curves for a particular parameterization that satisfies $z^h < z^*$. When $z_0 \in \left(z^h, z^*\right)$, there is an interior equilibrium that corresponds to the intersection of the two curves. When $z_0 > z^*$, the curves do not intersect and there is a corner equilibrium with the efficient asset price $\tilde{Q}_0 = 1$.

Figure 2 also illustrates how supply shocks can generate downward spirals in asset prices. The dashed line corresponds to the benchmark productivity realization, $z_0 = 1$, which leads to efficient asset prices. Starting from this level, a decline in productivity can
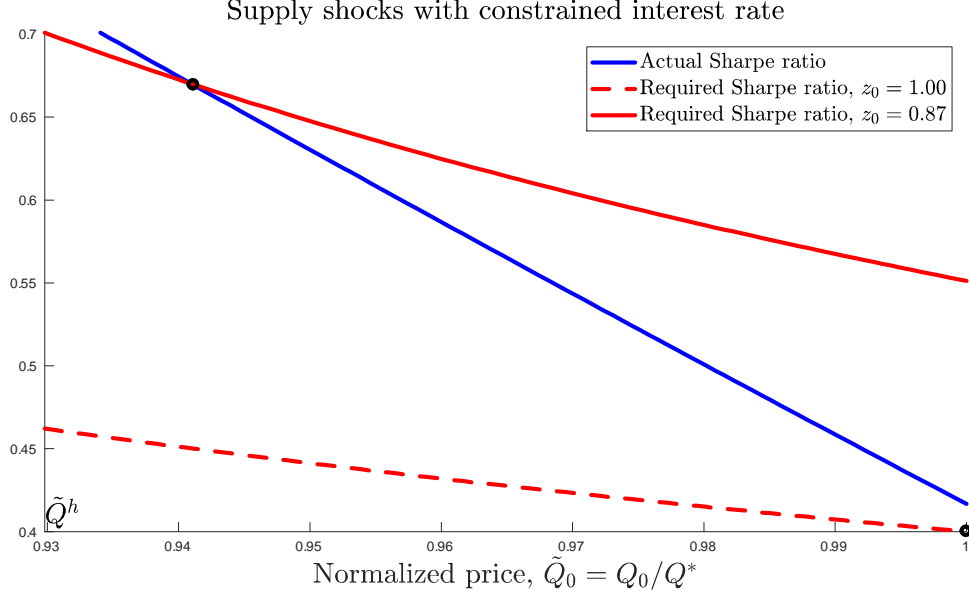
Figure 2: Effect of supply shocks when the interest rate is constrained—the case with a unique equilibrium.

lead to a substantially lower asset price per productivity (the intersection of the two solid lines). A decline in z_0 damages banks' balance sheets and increases the required Sharpe ratio. This leads to a reduction in asset prices—in order to increase the actual Sharpe ratio. The reduction in asset prices further damages banks' balance sheets and increases the required Sharpe ratio, which further reduces asset prices, and so on. In equilibrium, the Sharpe ratio rises more than the initial increase (captured by the vertical shift from the dashed to the solid red line). Consequently, the asset price falls considerably more than the direct effect of the negative supply shock.

To illustrate the second part of Proposition 1, consider parameters that satisfy $z^* < z^h$ and a shock that satisfies, $z_0 \in [z^*, z^h]$. Since $z_0 < z^h$, there is a bankruptcy equilibrium with the lowest asset price, $\tilde{Q}_0 = \tilde{Q}^h$. However, since $z_0 > z^*$, there is also an equilibrium with the efficient asset price, $\tilde{Q}_0 = 1$. Figure 3 illustrates these equilibria by plotting the required and the actual Sharpe ratio curves. The high and the low-price equilibria are marked with H and L, respectively. There is also an interior equilibrium that corresponds to the intersection of the two curves. (However, this equilibrium is unstable: small deviations would bring the equilibrium to either H or L).

To see the intuition for multiplicity, suppose we are currently at the high-price equilibrium H in Figure 3. In this equilibrium, high prices support banks' balance sheets, which raises the effective risk tolerance and reduces the required Sharpe ratio. This in turn keeps asset prices high. If prices fall sufficiently, then banks' balance sheets become substantially
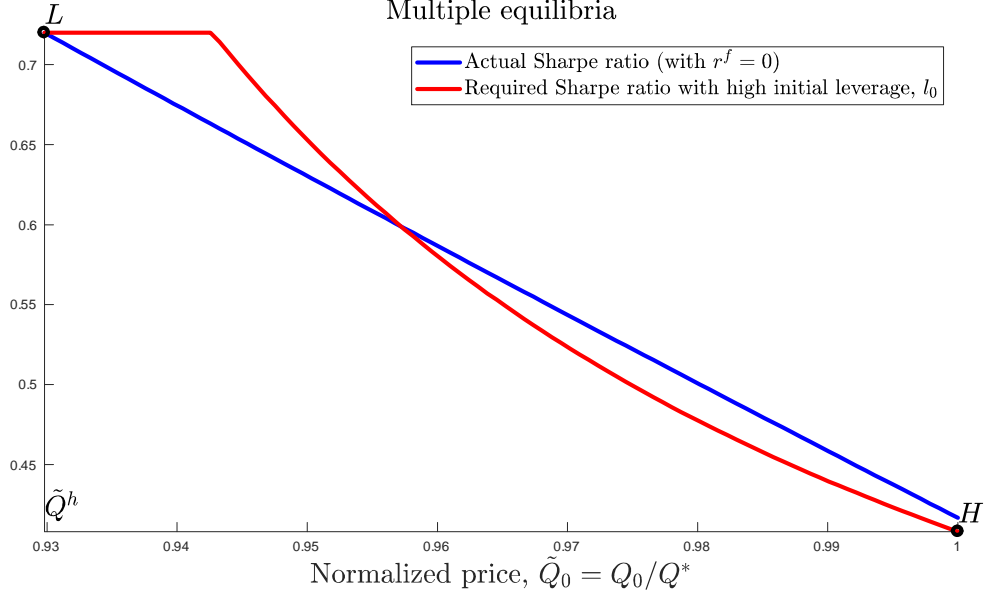
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Figure 3: Effect of supply shocks when the interest rate is constrained—the case with multiple equilibria.

weaker, which rapidly reduces the effective risk tolerance and raises the required Sharpe ratio. This in turn reinforces the large fall in asset prices and culminates in the low-price equilibrium L that features bankruptcy. As this discussion suggests, multiplicity is more likely when banks have greater leverage. In fact, the parameters used in Figure 3 are the same as those used in Figure 2, with the difference that we raise banks' initial leverage l_0 (and also adjust banks' risk tolerance τ^b to keep the benchmark risk tolerance $\tau_0(1)$ unchanged).

3. Large-scale Asset Purchases

The powerful downward spiral caused by the endogenous decline in risk tolerance suggests that policy interventions that absorb some of the risky assets during such events can be very powerful. To address this issue, we now introduce unconventional monetary policy in the form of large-scale asset purchases (LSAPs). This requires introducing a fiscal authority: even if the asset purchases are made by the central bank, the gains and losses from these positions ultimately accrue to the treasury. We merge the fiscal and monetary authorities into a third agent which we refer to as *the government* and denote by superscript g.

The government in our model is endowed with no resources in period 0 and a given amount of resources in period 1, denoted by $\eta^g z_1$. We think of $\eta^g z_1$ as the government's

future tax capacity. It can be "microfounded" by introducing a group of agents other than banks and households (e.g., the future generation) from which the government will be able to extract some taxes. We assume future tax capacity is proportional to future productivity, which simplifies the analysis but is not necessary for our results (in fact, making the government's tax capacity safer would strengthen our results). For simplicity the government starts with no assets. At the end of the period, it decides whether and how much to borrow, $b_0^g \geq 0$, and what fraction of the borrowed funds to invest in the risky asset, $\tilde{\omega}_0^g \geq 0$, with the residual fraction invested in the safe asset. In period 1, the government receives its tax revenues as well as the returns from its investments, pays back its debt, and spends the residual amount.

In the appendix, we show that the government's budget constraint can be rewritten analogously to the banks' and households' budget constraints. Specifically, the government's net wealth in period 0 is

$$a_0^g = z_0 Q_0 \eta^g, \tag{26}$$

and its *total* fraction of wealth invested in the market portfolio is

$$\omega_0^g = 1 + \frac{b_0^g \tilde{\omega}_0^g}{a_0^g}. \tag{27}$$

The government can be thought of as selling its future tax receipts and reinvesting the proceeds in the available assets. The value of its tax receipts is given by (26) and its net investment is given by (27). The requirements $b_0^g \geq 0$ and $\tilde{\omega}_0^g \geq 0$ translate into a requirement that the government takes a levered position, $\omega_0^g \geq 1$. The government is already fully exposed to the market portfolio through its future tax revenues, and it can further increase its exposure by borrowing and investing in risky assets.

Finally, the presence of the government changes the asset market clearing condition as follows [cf. (7)]:

$$\sum_{i \in \{b,h,g\}} \omega_0^i a_0^i = z_0 Q_0 \left(1 + \eta^g\right). \tag{28}$$

The right side illustrates that the government's tax capacity implicitly expands the supply of the market portfolio. The left side illustrates that the government also expands demand. Given a government portfolio choice $\omega_0^g \geq 1$, the definition of equilibrium generalizes in straightforward fashion. In the rest of the section, we characterize the equilibrium taking $\omega_0^g \geq 1$ as given and illustrate the comparative statics of LSAPs. We then introduce the government's preferences and characterize the optimal LSAP policy.

Equilibrium with large-scale asset purchases. Investors' optimality conditions are the same. Therefore much of the earlier analysis applies in this setting. Specifically, Eqs. $(10), (13)$, and (14) still hold. Using Eq. (28), we also obtain the analogue of the market clearing condition (12):

$$\alpha_0 \omega_0^b + (1 - \alpha_0) \omega_0^h + \eta^g \omega_0^g = 1 + \eta^g. \tag{29}$$

Combining these observations, we obtain the analogue of the risk balance condition (15):

$$\frac{\sigma (1 - \lambda)}{\tau_0 (z_0 Q_0 / Q^*)} = \frac{g - \log (Q_0) - r^f}{\sigma}, \tag{30}$$

where $\lambda = \eta^g (\omega_0^g - 1)$ and $\tau_0 (z)$ is given by the same expression as before [cf. (16)].

Eq. (30) illustrates that LSAPs effectively take some risk out of the market. Specifically, the risk balance condition is equivalent to an economy in which the risk is reduced by a fraction, $\lambda = \eta^g (\omega_0^g - 1)$. How much risk LSAPs remove depends on the government's tax capacity, η^g, and the riskiness of its portfolio, $\omega_0^g \geq 1$. When $\eta^g = 0$ or $\omega_0^g = 1$ the policy does not reduce risk and the risk balance condition (and the equilibrium) is the same as before. In subsequent analysis, we refer to λ as the size of the LSAP program.

In this context, first consider the equilibrium when the interest rate constraint does not bind. Substituting $Q_0 = Q^* = \exp (-\rho)$ into (30), we solve for the efficient interest rate [cf. Eq. (20)]:

$$r^{f*} = g + \rho - \frac{\sigma^2}{\tau_0 (z_0)} (1 - \lambda).$$

Hence, when the interest rate is not constrained, LSAPs do not affect asset prices, $z_0 Q^*$, or output, $y_0 = z_0$, but they translate into higher interest rates. As LSAPs take risk out of the market, they exert upward pressure on asset valuations and aggregate demand. Conventional monetary policy responds by raising the interest rate to keep asset prices and aggregate demand consistent with productivity.

We next consider the case in which the interest rate can be constrained and generalize Proposition 1. We assume the following analogue of (21):

$$\tau_0 (1) \geq \frac{\sigma^2}{\rho + g} > \frac{\tau^h}{1 - \lambda}. \tag{31}$$

As before, we also define two cutoff productivity levels, $z^h (\lambda), z^* (\lambda)$. Let

$$z^h (\lambda) = \frac{l_0}{\tilde{Q}^h (\lambda)} \quad \text{where} \quad \tilde{Q}^h (\lambda) = \exp \left(g + \rho - \frac{\sigma^2 (1 - \lambda)}{\tau^h} \right) < 1. \tag{32}$$

As before, $z^h(\lambda)$ is the cutoff productivity below which there is a bankruptcy equilibrium, and $\tilde{Q}^h(\lambda)$ is the normalized price in a bankruptcy equilibrium [cf. $(22-23)$]. Increasing λ increases the normalized price, $\tilde{Q}^h(\lambda)$, and decreases the cutoff, $z^h(\lambda)$: LSAPs increase the worst-case asset price level and shrink the set of productivity realizations that allow for bankruptcy.

Let $z^*(\lambda) \in (0, 1)$ denote the unique solution to

$$\frac{\tau_0(z^*)}{1 - \lambda} = \frac{\sigma^2}{\rho + g}. \tag{33}$$

As before, $z^*(\lambda)$ is the cutoff productivity above which there is a supply determined equilibrium with efficient prices [cf. (24)]. LSAPs expand the set of productivity realizations that allow for an efficient price equilibrium. The next result characterizes the equilibrium when it is unique and interior. The case with multiple equilibria is similar to Proposition 1 and discussed subsequently.

Proposition 2. *Consider the equilibrium with LSAPs, $\lambda = \eta^g(\omega_0^g - 1) \geq 0$, and conditions (31). Suppose $z_0 \in [z^h(\lambda), z^*(\lambda)]$ given the cutoffs in Eqs. $(32-33)$. There exists a unique equilibrium with an interior normalized price, $\tilde{Q}_0 \in (\tilde{Q}^h(\lambda), 1)$, that solves*

$$\frac{\sigma(1 - \lambda)}{\tau^h + \left(1 - \frac{l_0}{z_0 \tilde{Q}_0}\right) \kappa_0 (\tau^b - \tau^h)} = \frac{g + \rho - \log\left(\tilde{Q}_0\right)}{\sigma}. \tag{34}$$

The normalized price is increasing in the size of the LSAP program, $\frac{d\tilde{Q}_0}{d\lambda} > 0$.

Consider Figure 4, which illustrates Eq. (34). As the figure shows, LSAPs shift the required Sharpe ratio curve downward without affecting the actual Sharpe ratio curve. In equilibrium, this leads to a lower Sharpe ratio and a higher asset price. In fact, LSAPs have an *amplified* effect on the Sharpe ratio: the change in the equilibrium Sharpe ratio is much greater than the initial downward shift of the curve. As LSAPs increase asset prices, they improve banks' balance sheets, which further reduces the required Sharpe ratio and raises asset prices. Essentially, LSAPs help undo the downward spirals created by supply shocks illustrated in Figure 2.

LSAPs can have even more powerful effects when there are multiple equilibria. Figure 5 illustrates this by plotting the effect of LSAPs for parameters that lead to multiplicity. The solid red line illustrates the risk premium curve without policy, which leads to multiple equilibria (denoted by L and H in the figure). The dashed red line illustrates the effect
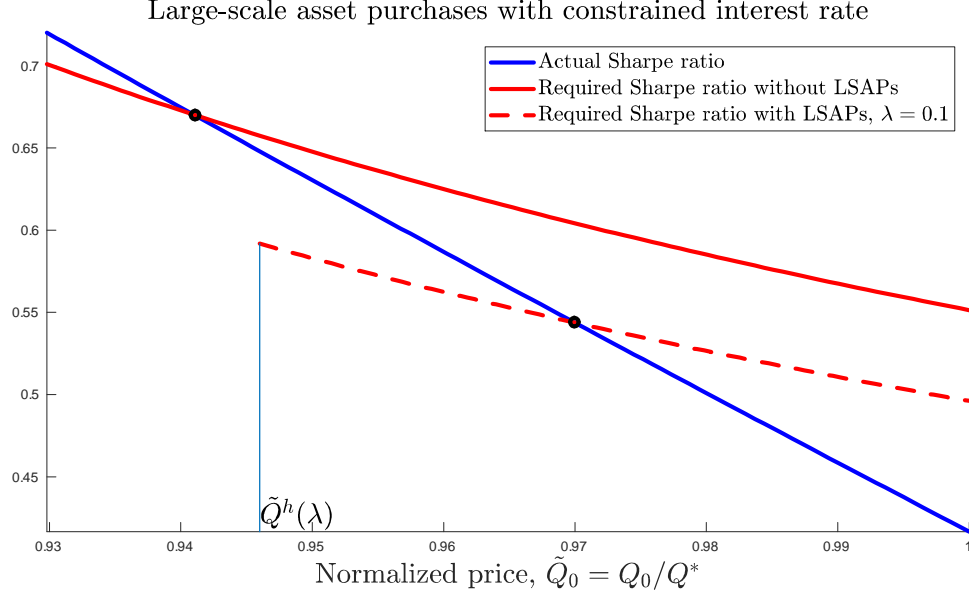
Figure 4: Effect of LSAPs when the interest rate is constrained—the case with a unique equilibrium.

of LSAPs with the same magnitude as in the previous case with a unique equilibrium. In this case, LSAPs eliminate the low-price equilibrium. By removing risk from the market, the policy reduces the required Sharpe ratio and increases asset prices, which triggers a virtuous spiral that culminates in the high-price equilibrium (denoted by H' in the figure).

Optimal large-scale asset purchases. We next introduce an objective function for the government and analyze the resulting optimal LSAP policy. Suppose the government chooses its portfolio weight, $\omega_0^g \geq 1$, to maximize:

$$\log\left(\tilde{Q}_0\right) - \frac{1}{2}e^{-\rho}\left(\eta^g \frac{1}{\tau^g}\left(\sigma\omega_0^g\right)^2 + \frac{1}{\tau_0\left(1\right)}\left(\sigma\left(1 - \eta^g\left(\omega_0^g - 1\right)\right)\right)^2\right). \tag{35}$$

The government's objective function features three terms. The first term, $\log\tilde{Q}_0 < 0$, captures the desire to close the output gap in period 0. In our model, this is equivalent to closing asset price gaps [cf. (10)]. The second term, $\frac{1}{\tau^g}\left(\sigma\omega_0^g\right)^2$, captures the risk in the government's portfolio. We assume the government has similar preferences as agents but with risk tolerance τ^g. The remaining term captures the residual risk in agents' portfolios. The government evaluates these risks according to the benchmark effective risk tolerance, $\tau_0\left(1\right)$—ignoring the changes in the effective risk tolerance due to the shock. This does not play an important role beyond simplifying the analysis (and it makes the case for LSAPs weaker). Finally, the government weights its own utility and the agents' utility with η^g
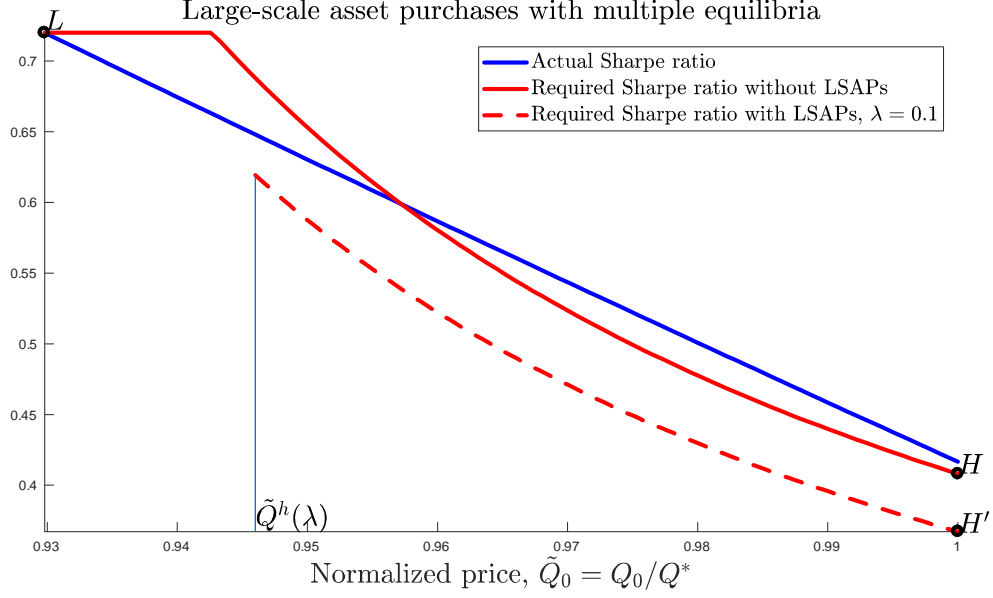
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Figure 5: Effect of LSAPs when the interest rate is constrained—the case with multiple equilibria.

and 1, respectively (the initial endowments of the market portfolio).[6]

We also assume the government is weakly less risk tolerant than the agents:

$$\tau^g \le \tau_0 (1). \tag{36}$$

This ensures that, if there was no demand recession, the government would *not* use LSAPs. That is, the reason for LSAPs in our model is *not* a financial friction. Instead, the government uses LSAPs to respond to the demand recession when it cannot cut interest rates. To see this, we rewrite (35) in terms of the size of the LSAP program, $\lambda = \eta^g (\omega_0^g - 1)$, to obtain the problem

$$\max_{\lambda \ge 0} \log \left(\tilde{Q}_0 (\lambda) \right) - \frac{1}{2} e^{-\rho} \sigma^2 \left(\eta^g \frac{1}{\tau^g} \left(1 + \frac{\lambda}{\eta^g} \right)^2 + \frac{1}{\tau_0 (1)} (1 - \lambda)^2 \right).$$

Suppose there is a unique and interior equilibrium price denoted by $\tilde{Q}_0 (\lambda)$ [cf. Proposition 2]. The condition for an optimum with a positive LSAP, $\lambda > 0$, is then given by

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[6]The functional forms in (35) reflect agents' utility functions and they are motivated by the standard Pareto criterion. In Appendix A.2, we consider a slight variant of the model in which banks and the households are merged into a single agent, "the market," with exogenous risk tolerance τ^m. We derive a constrained Pareto problem and show that it features the objective in (35) with $\tau^m = \tau_0 (1)$.

$$\frac{1}{\tau^g}\left(1 + \frac{\lambda}{\eta^g}\right)\sigma^2 = \frac{1}{\tau_0(1)}(1-\lambda)\sigma^2 + \frac{1}{e^{-\rho}}\frac{\tilde{Q}_0'(\lambda)}{\tilde{Q}_0(\lambda)}. \tag{37}$$

Eq. (37) says that the government's optimal tolerance-adjusted portfolio risk (the left side) is the sum of the market's tolerance-adjusted risk and the marginal effect of the LSAPs on asset prices, $\frac{\tilde{Q}_0'(\lambda)}{\tilde{Q}_0(\lambda)}$. If the latter was zero, then the corner solution $\lambda = 0$ would be optimal since the government is relatively less risk tolerant. When the economy is in a demand recession, the latter term is strictly positive, $\frac{\tilde{Q}_0'(\lambda)}{\tilde{Q}_0(\lambda)} > 0$ [cf. Proposition 2], so the government might find it optimal to use LSAPs.

Eq. (37) also suggests that the size of the optimal LSAP satisfies intuitive comparative statics (which we verify in numerical simulations). The optimal LSAP is increasing in the government's risk tolerance, $\tau^g > 0$, and its tax capacity, $\eta^g > 0$. Greater capacity helps because it enables the government to achieve the same impact on financial markets with a smaller impact on its own risk exposure.

More subtly, we find that (as long as banks are not bankrupt under the optimal LSAP) the government tends to engage in larger LSAP when the supply shock is more severe (lower z_0) and when the private sector initially has greater leverage (greater l_0). Figure 6 illustrates these results for the parameters in our earlier analysis (see Figures 2 and 4). We set the government's risk tolerance to be the same as households' risk tolerance, $\tau^g = \tau^h$, so it is quite costly for the government to absorb risk. Nonetheless, the government chooses to use LSAPs. The left panel shows that increasing the severity of the shock increases the size of the optimal LSAP. The right panel shows that increasing banks' initial leverage has the same effect. In this panel, as we increase l_0 we also adjust banks' risk tolerance to keep the effective benchmark risk tolerance $\tau_0(1)$ unchanged (which leads to a more meaningful comparison).

To see the intuition, consider how the severity of the supply shock affects the equilibrium and the government's trade-off. Our earlier Figure 2 shows that lower z_0 increases the steepness of the required risk premium curve. When the required risk premium curve is steeper, downward spirals are more severe. LSAPs then have a greater impact on asset prices because they help undo these spirals [cf. Figure 4]. Consequently, lower z_0 increases the marginal benefit from the LSAPs, $\frac{\tilde{Q}_0'(\lambda)}{\tilde{Q}_0(\lambda)}$, which implies larger optimal LSAPs. Likewise, greater initial l_0 exacerbates the downward spirals and increases the marginal benefit from LSAPs.

As this intuition suggests, these results apply for an interior equilibrium but they might not hold for a corner equilibrium in which banks are bankrupt. With bankruptcy, there are parameters where *improving* productivity z_0 *increases* the optimal LSAP. This
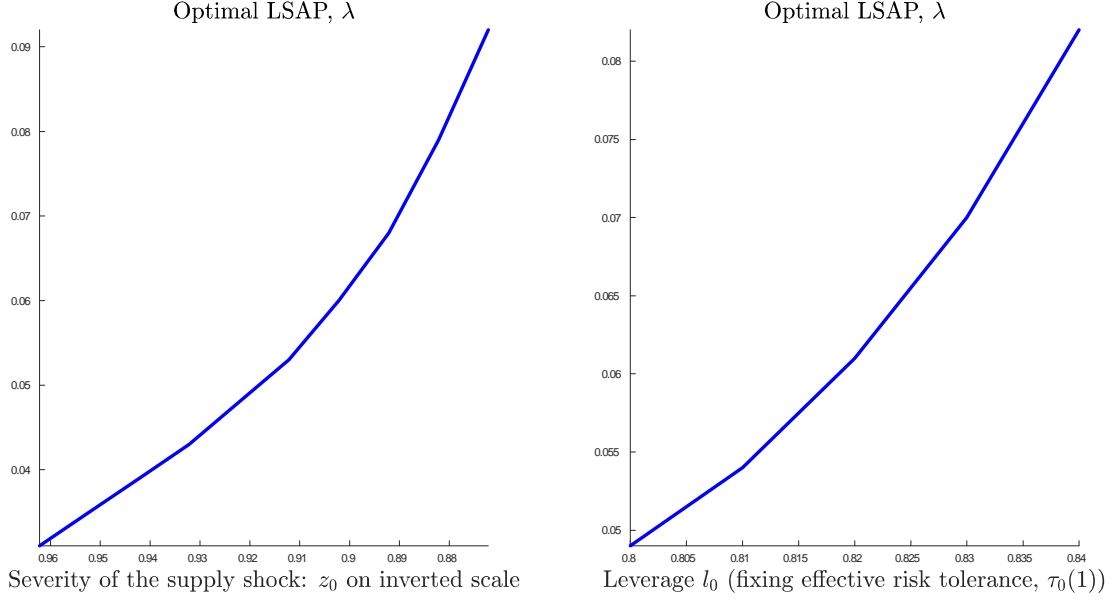
Figure 6: Optimal LSAP as a function of productivity, z_0 (left panel, inverted scale) and banks' initial leverage, l_0 (right panel).

happens when the government finds it too costly to save the banks via a large enough LSAP program. As z_0 improves, the government at some point finds it optimal to save the banks, which induces a discrete upward jump in the optimal LSAP. Likewise, when banks are bankrupt, *decreasing* their initial leverage might *increase* the optimal LSAP.

4.   Final Remarks

In this paper we show how asset price spirals and aggregate demand can amplify supply shocks when economic agents have heterogeneous risk tolerance. As supply drops, so do asset prices and the wealth share of risk tolerant agents. Thus, the "representative agent" becomes less risk tolerant and demands a higher Sharpe ratio from the market to hold its risky assets. With unconstrained monetary policy, a cut in interest rates is the most effective mechanism to increase the market's Sharpe ratio. If the monetary authority cannot cut interest rates, asset prices drop further and drag down aggregate demand and the wealth share of risk tolerant agents, triggering a downward spiral.

This perspective hints that effective risk-centric policies align the required and actual Sharpe ratios at asset price levels consistent with efficient output. In the body of the paper we discussed one such policy: LSAPs. This policy works by reducing the supply of risk that economic agents need to absorb in equilibrium, which puts upward pressure on asset prices, which increases the risk tolerant agents' wealth share, which increases effective

risk tolerance, and so on. This multiplier is stronger if the economy is more unstable at the outset, which is a direct function of the amount of leverage in the system. Our analysis is highly supportive of the unprecedented (in terms of size and speed) asset market interventions by the Fed and other major central banks around the world in response to the financial distress caused by the Covid-19 shock, and it highlights the importance of targeting assets held by levered investors. Importantly, the rationale for this policy in our framework is not to protect "the financial pipeline," however important this may be, but to boost aggregate demand when conventional monetary policy is constrained.

More loosely, one could think of other policy mechanisms to influence the required and actual Sharpe ratios. For example, loosening capital requirements is likely to increase effective risk tolerance and hence reduce the required Sharpe ratio. By the same token, any public guarantee or put policy that reduces perceived volatility is likely to reduce the gap between required and actual Sharpe ratios at any given asset price level. We will explore some of these policies in future work.

Finally, we chose to illustrate the key results in a simple two-period model. However, we have explored the continuous time version of this model along the lines of Caballero and Simsek (forthcoming) and found similar mechanisms at work, but with stronger feedbacks as current negative shocks feed into future demand recessions, which feed back into asset prices. We will add these results in future versions of this paper.

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A. Appendix: Omitted derivations

This appendix presents the derivations and proofs omitted from the main text. We start by presenting the details of the baseline case without LSAPs. We then consider the case with LSAPs. Throughout, recall that the market portfolio is the claim to all output. Combining Eqs. (1) and (2), the return on the market portfolio at date 1 is also log normally distributed, that is,

$$r(z_0, z_1) = \log\left(\frac{z_1}{z_0 Q_0}\right) \sim N\left(\log\frac{\overline{z}_1}{z_0} - \log(Q_0) - \frac{\sigma^2}{2}, \sigma^2\right). \tag{A.1}$$

For most of our analysis, we also assume $\log\frac{\overline{z}_1}{z_0} = g$, which further simplifies this expression [cf. Eq. (19)].

A.1. Baseline model without policy

Most of the analysis is provided in the main text. Here, we formally state the agents' problem that incorporates the log-Normal approximation. We also derive the optimality conditions. We then complete the characterization of equilibrium and prove Proposition 1.

Approximate portfolio problem and optimality conditions. Without an approximation, type i agents would solve the following problem,

$$u_0^{i,exact}(\tilde{a}_0^i) = \max_{c_0, a_0, \omega} \log c_0 + e^{-\rho} \log u_1^i \tag{A.2}$$

$$\text{where } u_1^i = \left(E\left[c_1(z_0, z_1)^{(\tau^i - 1)/\tau^i}\right]\right)^{\tau^i/(\tau^i - 1)}$$

$$\text{s.t.} \quad c_0 + a_0 = \tilde{a}_0^i$$

$$\text{and} \quad c_1(z_0, z_1) = a_0\left(\omega \exp(r(z_0, z_1)) + (1 - \omega)\exp(r^f)\right). \tag{A.3}$$

Here, $c_1(z_0, z_1)$ denotes total financial wealth, which equals consumption (since the economy ends at date 1). Note that the agent has Epstein-Zin preferences with EIS coefficient equal to one and the RRA coefficient equal to $1/\tau^i > 0$. The case with $\tau^i = 1$ is equivalent to time-separable log utility. Agents' initial endowments, a_0^i, are given by (3).

In view of the Epstein-Zin functional form, agents can be thought of as solving the intertemporal problem,

$$u_0^i(\tilde{a}_0^i) = \max_{a_0} \log(\tilde{a}_0^i - a_0) + e^{-\rho} \log(R^{CE,i} a_0). \tag{A.4}$$

Here, $R^{CE,i}$ denotes investors' certainty-equivalent portfolio return per dollar. Absent an approximation, it would correspond to the solution to the following portfolio optimization problem:

$$R^{CE,i,exact} = \max_{\omega} \left(E \left[(R^p(z_0, z_1))^{(\tau^i - 1)/\tau^i} \right] \right)^{\tau^i/(\tau^i - 1)} \tag{A.5}$$

$$\text{and } R^p(z_0, z_1) = \omega \exp(r(z_0, z_1)) + (1 - \omega) \exp(r^f).$$

The variable, $R^p(z_0, z_1)$, denotes the realized portfolio return per dollar.

In our analysis, we assume that agents choose portfolios (and evaluate the resulting certainty-equivalent return, $R^{CE,i}$) by solving the following approximate portfolio problem:

$$\log R^{CE,i} - r^f = \max_{\omega} \omega\pi - \frac{1}{2}\frac{1}{\tau^i}\omega^2\sigma^2 \tag{A.6}$$

$$\text{where } \pi = E[r(z_0, z_1)] + \frac{\sigma^2}{2} - r^f.$$

Here, π denotes the risk premium on the market portfolio and σ is its standard deviation (measured in log returns). The problem says that the agent trades off its portfolio mean (in excess of the risk-free rate), $\omega\pi$, with its portfolio variance, $\omega^2\sigma^2$. This approximation becomes exact if the portfolio return follows a log-Normal distribution. In general, this is not the case and it holds only approximately. This approximation works well for calibrations with relatively short time horizons and it becomes exact in continuous time. The approximation is widely used in the literature (see Campbell and Viceira (2002)).

The first order condition for problem $(A.4)$ implies Eq. (5) in the main text. That is, regardless of her certainty-equivalent portfolio return, the investor consumes and saves a constant fraction of her lifetime wealth.

The first order condition for problem $(A.6)$ implies Eq. (6) in the main text.

Characterization of equilibrium. We characterize the equilibrium and prove Proposition 1. We first characterize the equilibrium in terms of an auxiliary function. Consider the function:

$$F\left(\tilde{Q}_0; z_0\right) = \frac{\sigma^2}{\overline{\tau}_0 - \frac{\kappa_0 l_0 \left(\tau^b - \tau^h\right)}{z_0 \tilde{Q}_0}} + \log\left(\tilde{Q}_0\right) - (g + \rho) \tag{A.7}$$

$$\text{where } \overline{\tau}_0 = \tau^h + \kappa_0 \left(\tau^b - \tau^h\right).$$

This function is defined over the domain $\tilde{Q}_0 \in \left(\underline{Q}_0, \infty\right)$, where $\underline{Q}_0 = \frac{\kappa_0 l_0 \left(\tau^b - \tau^h\right)/z_0}{\overline{\tau}_0}$. Eq. (15) implies that every interior equilibrium, $\tilde{Q}_0 \in \left(\tilde{Q}_0^h, 1\right)$, corresponds to a zero of this function. Conversely, every zero of the function that falls in the range, $\tilde{Q}_0 \in \left(\tilde{Q}_0^h, 1\right)$, corresponds to an interior equilibrium. The zeros that fall outside this range do not correspond to an equilibrium. Finally, there is a corner equilibrium with $\tilde{Q}_0 = 1$ (and $r^f \geq 0$) iff $F\left(1; z_0\right) \leq 0$; and there is a corner equilibrium with $\tilde{Q}_0 = \tilde{Q}_0^h$ (and bankruptcy) iff $F\left(\tilde{Q}_0^h; z_0\right) \geq 0$.

We next establish some properties of the auxiliary function that facilities the proof. Consider the monotone change of variables:

$$\tau_0 = \overline{\tau}_0 - \frac{\kappa_0 l_0 \left(\tau^b - \tau^h\right)}{z_0 \tilde{Q}_0} \quad \Leftrightarrow \quad \tilde{Q}_0 = \frac{\kappa_0 l_0 \left(\tau^b - \tau^h\right)/z_0}{\overline{\tau}_0 - \tau_0}. \tag{A.8}$$

In terms of the new variable, the auxiliary function corresponds to the transformed function:

$$f\left(\tau_0\right) \equiv \frac{\sigma^2}{\tau_0} - \log\left(\overline{\tau}_0 - \tau_0\right) + \log l_0 - \log z_0 - \left(g + \rho\right). \tag{A.9}$$

This function has the domain $\tau_0 \in \left(0, \overline{\tau}_0\right)$, and it is strictly convex, that is:

$$f''\left(\tau_0\right) = \frac{2\sigma^2}{\tau_0^3} + \frac{1}{\left(\overline{\tau}_0 - \tau_0\right)^2} > 0$$

The function also satisfies $\lim_{\tau_0 \to 0} f\left(\tau_0\right) = \lim_{\tau_0 \to \infty} f\left(\tau_0\right) = \infty$. These observations imply that the zeros of the transformed function $f\left(\cdot\right)$ have the same characteristics as an upward-pointing parabola. The original function $F\left(\cdot; z_0\right)$ adopts the same characteristics. In particular, the function either does not have any (interior) zero:

$$F\left(\tilde{Q}_0; z_0\right) \geq 0 \text{ for } \tilde{Q}_0 \in \left(\underline{Q}_0, \infty\right), \tag{A.10}$$

or it has exactly two interior zeros:

$$F\left(Q_0^1; z_0\right) = F\left(Q_0^2; z_0\right) = 0 \text{ for } \underline{Q}_0 < Q_0^1 < Q_0^2 \tag{A.11}$$
$$\text{with } F\left(\tilde{Q}_0; z_0\right) < 0 \text{ for } \tilde{Q}_0 \in \left(Q_0^1, Q_0^2\right) \text{ and } F\left(\tilde{Q}_0; z_0\right) > 0 \text{ otherwise.}$$

Proof of Proposition 1. Consider the first part that concerns the case, $z_0 > z^h = \frac{l_0}{\tilde{Q}^h}$

[cf. (23)]. This condition implies the auxiliary function in $(A.7)$ satisfies:

$$F\left(\tilde{Q}_0^h; z_0\right) = \frac{\sigma^2}{\tau_0\left(z_0\tilde{Q}_0^h\right)} + \log\left(\tilde{Q}_0^h\right) - (g+\rho)$$

$$< \frac{\sigma^2}{\tau^h} + \log\left(\tilde{Q}_0^h\right) - (g+\rho) = 0. \tag{A.12}$$

Here, the inequality follows since $z_0 > z^h = \frac{l_0}{\tilde{Q}^h}$ implies $\tau_0\left(z_0\tilde{Q}_0^h\right) > \tau\left(l_0\right) = \tau^h$. The equality follows from the definition of $\tilde{Q}_0^h$. This rules out the corner equilibrium with $\tilde{Q}_0 = \tilde{Q}_0^h$. Combining this observation with Eq. $(A.11)$ also implies that we must have the case $(A.11)$ with $\tilde{Q}_0^h$ falling between the two zeros. This in turn implies there is a unique equilibrium that depends on the sign of $F\left(1; z_0\right)$. When $F\left(1; z_0\right) > 0$, there is an interior equilibrium with $\tilde{Q}_0 \in \left(\tilde{Q}_0^h, 1\right)$. When $F\left(1; z_0\right) \leq 0$, there is a corner equilibrium with $\tilde{Q}_0 = 1$. Note also that $F\left(1; z_0\right) = \frac{\sigma^2}{\tau_0(z_0)} - (g+\rho)$ implies that the condition, $F\left(1; z_0\right) > 0$, is equivalent to $z_0 < z^*$ from the definition of z^* [cf. (24)]. This proves that there is a unique interior equilibrium when $z_0 < z^*$ (and $z_0 > z^h$) and there is a unique corner equilibrium when $z_0 \geq z^*$ (and $z_0 > z^h$).

Next consider the comparative statics of the interior equilibrium with respect to z_0. Note that $F\left(\tilde{Q}_0; z_0\right)$ is decreasing in z_0. Therefore, greater z_0 shifts $F\left(\tilde{Q}_0; z_0\right)$ downward, which increases the (greater) zero of the function that corresponds to the equilibrium. This establishes $\frac{d\tilde{Q}_0}{dz_0} > 0$ and completes the proof of the first part.

Next suppose $z_0 < z^h = \frac{l_0}{\tilde{Q}^h}$. We have the opposite of $(A.12)$, which implies that there is a corner equilibrium with $\tilde{Q}_0 = \tilde{Q}_0^h$. In this case, there can also be other equilibria. To see this, consider $z_0 \in \left(z^*, z^h\right)$ (assuming the interval is nonempty). Then, we have:

$$F\left(1; z_0\right) = \frac{\sigma^2}{\tau_0\left(z_0\right)} - (g+\rho) < \frac{\sigma^2}{\tau_0\left(z^*\right)} - (g+\rho) = 0. \tag{A.13}$$

Here, the inequality follows since $z_0 > z^*$ and the equality follows from the definition of z^*. This implies that there is a corner equilibrium with $\tilde{Q}_0 = 1$. In particular in this case $\tilde{Q}_0 = \tilde{Q}_0^h$ and $\tilde{Q}_0 = 1$ are both corner equilibria. This completes the proof of the proposition. □

A.2. Model with large-scale asset purchases

We next present the details of the extended analysis with LSAPs. We first describe the government's budget constraint and derive Eqs. (26) and (27). We then complete

the characterization of the equilibrium and prove Proposition 2. Finally, we provide a rationale for the government's objective function (35).

Government's budget constraints. The government is endowed with some income (tax receipts) in period 1 given by $z_1 \eta^g$. At the end of period 0, the government decides how much to borrow, $b_0^g \geq 0$, and what fraction of the borrowed funds to invest in the risky asset, $\tilde{\omega}_0^g \geq 0$, with the residual fraction invested in the safe asset. In period 1, the government collects the tax receipts and the return on its investments, pays back its debt, and spends the residual. Its budget constraint in period 1 can be written as:

$$c_1^g = z_1 \eta^g + b_0^g \left(\tilde{\omega}_0^g \exp\left(r\left(z_0, z_1\right)\right) - \tilde{\omega}_0^g \exp\left(r^f\right) \right), \tag{A.14}$$

where c_1^g denotes government spending in period 1.

We next rewrite the budget constraint in $(A.14)$ to make it parallel to the agents' budget constraint in $(A.3)$. Eq. (2) implies $z_1 = z_0 Q_0 \exp\left(r\left(z_0, z_1\right)\right)$. Substituting this into the budget constraint, we obtain:

$$
\begin{aligned}
c_1^g &= z_0 Q_0 \eta^g \exp\left(r\left(z_0, z_1\right)\right) + b_0^g \tilde{\omega}_0^g \exp\left(r\left(z_0, z_1\right)\right) - b_0^g \tilde{\omega}_0^g \exp\left(r^f\right) \\
&= a_0^g \left(\left(1 + \frac{b_0^g \tilde{\omega}_0^g}{a_0^g}\right) \exp\left(r\left(z_0, z_1\right)\right) - \frac{b_0^g \tilde{\omega}_0^g}{a_0^g} \exp\left(r^f\right) \right) \\
&= a_0^g \left(\omega_0^g \exp\left(r\left(z_0, z_1\right)\right) + \left(1 - \omega_0^g\right) \exp\left(r^f\right) \right) \\
\text{where } a_0^g &= z_0 Q_0 \eta^g \text{ and } \omega_0^g = 1 + \frac{b_0^g \tilde{\omega}_0^g}{a_0^g}.
\end{aligned}
$$

The second line defines and substitutes the effective wealth of the government, a_0^g, and the third line defines and substitutes the effective portfolio weight, ω_0^g. This establishes Eqs. (26) and (27) in the main text. The government can be effectively thought of as starting with the present discounted value of its tax receipts and choosing a weight on the market portfolio that reflects its implicit holding of risky tax receipts as well as its additional investments.

Characterization of equilibrium with LSAPs. Consider the analogue of the function $(A.7)$ that incorporates LSAPs:

$$
F\left(\tilde{Q}_0; z_0, \lambda\right) = \frac{\sigma^2 \left(1 - \lambda\right)}{\overline{\tau}_0 - \frac{l_0 \kappa_0 \left(\tau^b - \tau^h\right)}{z_0 \tilde{Q}_0}} + \log\left(\tilde{Q}_0\right) - \left(g + \rho\right) \tag{A.15}
$$

$$
\text{where } \overline{\tau}_0 = \tau^h + \kappa_0 \left(\tau^b - \tau^h\right).
$$

Every interior equilibrium, $\tilde{Q}_0 \in \left(\tilde{Q}_0^h(\lambda), 1 \right)$, corresponds to a zero of this function. Conversely, any zero of the function that falls in the interior range, $\tilde{Q}_0 \in \left(\tilde{Q}_0^h(\lambda), 1 \right)$, corresponds to an equilibrium. The zeros that fall outside this range do not correspond to an equilibrium. There is a corner equilibrium with $\tilde{Q}_0 = 1$ iff $F(1; z_0, \lambda) \leq 0$; and there is a corner equilibrium with $\tilde{Q}_0 = \tilde{Q}_0^h$ iff $F\left(\tilde{Q}_0^h; z_0, \lambda \right) \geq 0$. Finally, the function $F\left(\tilde{Q}_0; z_0, \lambda \right)$ satisfies the same property that we established for the special case with $\lambda = 0$: one of cases $(A.10)$ and $(A.11)$ holds.

Proof of Proposition 2. Suppose $z^h(\lambda) < z^*(\lambda)$ and consider a shock $z_0 \in \left(z^h(\lambda), z^*(\lambda) \right)$. Following the same steps as in Proposition 1, there exists a unique equilibrium that corresponds to the (greater) zero of the function, $F\left(\tilde{Q}_0; z_0, \lambda \right)$, that falls in the range, $\tilde{Q}_0 \in \left(\tilde{Q}^h(\lambda), 1 \right)$. Consider the comparative statics with respect to the size of the LSAP, λ. Eq. $(A.15)$ implies that increasing λ shifts the function, $F\left(\tilde{Q}_0; z_0, \lambda \right)$, downward. This increases the (greater) zero and raises the equilibrium price, that is, $\frac{d\tilde{Q}_0}{d\lambda} > 0$. $\square$

Government's objective function. We next provide a rationale for the functional form assumptions in the government's objective function (35). For concreteness, suppose there is a future generation of agents (born in period 1) that are the residual claimant from the government's positions and thus consume c_1^g. The future generation's utility function is similar to the other agents' (constant relative risk aversion) with risk tolerance τ^g. We also simplify the setup by merging the other agents (banks and households) into a single agent, which we refer to as the market, with risk tolerance τ^m. Finally, suppose the government assigns the relative Pareto weights η^g and 1 to the future generation and the market—chosen to match their relative endowments. We set up a constrained Pareto problem in which the government's only policy tool is to choose $\omega_0^g \geq 1$. We we show that the objective function is the same as (35) with $\tau^m = \tau_0(1)$.

First consider the characterization of equilibrium for a given $\omega_0^g \geq 1$. Suppose $\tau^m < \frac{\sigma^2 \left(1 - \eta^g \left(\omega_0^g - 1 \right) \right)}{\rho + g}$, which implies there is a demand recession in period 0 despite the LSAP (the other case is similar). Following the same steps in Section 3, the equilibrium in period 0 features $r^f = 0$ and the asset price [cf. (34)]:

$$\log \left(\tilde{Q}_0 \right) = g + \rho - \frac{\sigma^2 \left(1 - \eta^g \left(\omega_0^g - 1 \right) \right)}{\tau^m} < 1. \tag{A.16}$$

As before, the asset price is increasing in ω_0^g. The market clearing condition is:

$$\omega_0^m + \eta^g \omega_0^g = 1 + \eta^g. \tag{A.17}$$

We next calculate the market's equilibrium utility. Using Eqs. $(A.4)$, $(A.6)$, and (5), we have:

$$
\begin{aligned}
u_0^m &= \log\left(\frac{z_0 Q_0}{1 + e^{-\rho}}\right) + e^{-\rho}\left(\log\left(R^{CE,m}\right) + \log\left(\frac{e^{-\rho} z_0}{1 + e^{-\rho}}\right) + \log Q_0\right) \\
&= \tilde{u}_0^m + \log\left(Q_0\right) + e^{-\rho}\left(\omega_0^m\left(g - \log Q_0\right) + \log Q_0 - \frac{1}{2}\frac{1}{\tau^m}\left(\omega_0^m\right)^2 \sigma^2\right).
\end{aligned}
$$

The second line substitutes the equilibrium interest rate, $r^f = 0$, and the equilibrium return on the market portfolio, $E\left[r\left(z_0, z_1\right)\right] + \frac{\sigma^2}{2} = g - \log Q_0$ [cf. Eq. (2)]. It also collects the exogenous terms into $\tilde{u}_0^m$.

Next consider the future generation. They have the same utility function as the other agents but with risk tolerance τ^g. Their exact utility function is given by [cf. $(A.4 - A.5)$]:

$$
\begin{aligned}
u_1^{g,exact} &= e^{-\rho}\log\left(R^{CE,g} a_0^g\right) \text{ with } a_0^g = z_0 Q_0 \eta^g, \\
\text{where } R^{CE,g,exact} &= \left(E\left[\left(R^p\left(z_0, z_1\right)\right)^{(\tau^g - 1)/\tau^g}\right]\right)^{\tau^g/(\tau^g - 1)} \\
\text{and } R^p\left(z_0, z_1\right) &= \left(\omega_0^g \exp\left(r\left(z_0, z_1\right)\right) + \left(1 - \omega_0^g\right)\exp\left(r^f\right)\right).
\end{aligned}
$$

Applying the log-Normal approximation (similar to the other agents), we write this as [cf. $(A.6)$]:

$$
\begin{aligned}
u_1^g &= e^{-\rho}\left(\log\left(R^{CE,g}\right) + \log\left(z_0 \eta^g\right) + \log Q_0\right) \\
&= \tilde{u}_1^g + e^{-\rho}\left(\omega_0^g\left(g - \log Q_0\right) + \log Q_0 - \frac{1}{2}\frac{1}{\tau^g}\left(\omega_0^g\right)^2 \sigma^2\right).
\end{aligned}
$$

Where the second line substitutes for the equilibrium returns and collects the exogenous terms into $\tilde{u}_1^g$.

Aggregating the market's and the future generation's utility with weights 1 and η^g,

the government's objective function is:

$$
\begin{aligned}
U_0^g &= \tilde{U}_0^g + \log\left(Q_0\right) + e^{-\rho}\left(\begin{array}{c} -\left(\omega_0^m + \eta^g\omega_0^g\right)\log Q_0 + \left(1+\eta^g\right)\log Q_0 \\ -\frac{1}{2}\frac{1}{\tau^m}\left(\omega_0^m\right)^2\sigma^2 - \eta^g\frac{1}{2}\frac{1}{\tau^g}\left(\omega_0^g\right)^2\sigma^2 \end{array}\right) \\
&= \tilde{U}_0^g + \log\left(Q_0\right) - \frac{1}{2}e^{-\rho}\sigma^2\left(\frac{1}{\tau^m}\left(\omega_0^m\right)^2 + \eta^g\frac{1}{\tau^g}\left(\omega_0^g\right)^2\right) \qquad (A.18)
\end{aligned}
$$

Here, we have collected the exogenous terms into $\tilde{U}_0^g$. The second line uses the market clearing condition $(A.17)$ to simplify the expression.

Combining Eqs. $(A.18)$ and $(A.16)$ (and using $\tilde{Q}_0 = Q_0/Q^*$), the government's constrained Pareto problem is:

$$
\max_{\omega_0^g}\log\left(\tilde{Q}_0\right) - \frac{1}{2}e^{-\rho}\sigma^2\left(\frac{1}{\tau^m}\left(\omega_0^m\right)^2 + \eta^g\frac{1}{\tau^g}\left(\omega_0^g\right)^2\right),
$$

$$
\text{where } \log\left(\tilde{Q}_0\right) = g + \rho - \frac{\sigma^2\left(1-\eta^g\left(\omega_0^g-1\right)\right)}{\tau^m}.
$$

This problem is similar to the one we solve in the main text. In particular, the objective function is the same as in (35) after replacing $\tau^m = \tau_0\left(1\right)$, which provides a rationale for the functional forms.[7]

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[7]This analysis also clarifies the role of the Pareto weights, 1 and η^g. These weights are chosen to match the agents' initial endowments, which ensures that the pecuniary externalities generated by the changes in the price of the market portfolio "net out" [see $(A.18)$].